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Glide Effects on Low Speed Unmanned Aerial Vehicles with Ice Formation

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Abstract

This paper builds on the research that was previously presented and defended at a conference and adds to the depth of glide characteristics and aerodynamics. The basic assumed theory of low speed flight is used as the starting point and determined if this holds true for low speed. Traditionally, flight has always been about achieving faster and high flight operations. Unmanned Aerial Vehicles (UAV) are not typically designed to fly fast, the construction and power units are limiting, added to the lack of complexity in propulsion systems prevents high speeds in most cases. Here, aerodynamic data for Ice on leading edge and top surfaces were analysed, the characteristics obtained and thus the limitations. Furthermore, the influences of this to for unmanned aerial vehicles when subjected to surface environmental conditions such as ice on the leading edge and upper surface. Tested in a wind tunnel to see how theory compares with practice at various speed including take-off, landing and operational applications where head winds substantially alter parameters and this is data used to determine glide paths for landing. It also recommends design and operational changes to limited situations.

Keywords: Aerodynamics, low speed flight, unmanned vehicles and environmental influences.

1. Introduction

The majority of modern commercial aircraft have systems incorporated to detect ice and even prevent or remove ice, these are discussed later in this paper. Unmanned Aerial vehicles (UAV) are typically low cost, simple and do not have such additions or luxuries for detection and removal of ice. Any ice formed will have negative influences on the flight characteristics and stability of all phases of flight, particularly take-off and landing. Of course, ice prevention or removal before take-off can be solved; when in flight this is another problem. Design and calculations for UAVs are based on established theory and knowledge that has been accepted for many years. These parameters can only be achieved when an aeroplane is behaving according to the design specified and within its normal operations. In the history of aviation there has not been a need to focus on low speed flight as most aircraft are designed to fly at speeds far in excess of stall conditions.

With low speed aircraft the need for speed does not always apply anymore, For example, can theory predict what will happen from flight control movements through the operator for any situation? Environmental effects are common and have been experienced by anyone that has ever flown. Turbulence is one that is classified by most as a problem is in fact an air pocket where a change in the air density happens on a localized level, (O'Connor 1994). The majority of passengers that have flown in the very north or south of the world will have experienced delays from extremes in the weather. Aircraft that need to have ice removed or ice prevention spray to stop ice building up during the critical part of take-off. Theories have been known by designers and how they are to influence a buildup of ice on wings is known and classified in three ways, see figure 1, (Yodice, John S. (August 2005). First, Smooth, ideal as shown in blue, an ice prevention spray will keep



smooth during the critical phase of flight. Leading edge, upper surface are the critical ones, when combined and even worse result would be obtained, not addressed in this research paper to any depth, (Bertin J and Cummings R. (2008). The effects of these can be clearly seen with the lift v. α_{max} graph from figure 1. Lift is drastically reduced for each type of ice as is the maximum lift value. Leading edge ice is by far the most influential factor and many reported crashes have happened from this cause. When it is combined with ice on the upper surface this is compounded to make any take-off dangerous. When ice has formed, as the aircraft is parked on the ground, a pilot can decide to control or delay take-off. In flight when needing to land the same applies, (FAA, 2012). There are various mechanical and electrical methods of heating and ice prevention that are used depending on the aeroplane, (McAndrew IR, Navarro E and Witcher K, 2015). Figure 3 shows one such leading edge design method, here an inflatable rubber based material that forms a leading edge profile can be incorporated and as the expansion of this rubber tube expands it breaks the ice and simply falls away, (Ison *et al*, 2013)

This may be impractical for large aeroplane, it is a feasible and easy solution for smaller and unmanned ones. Nevertheless, it carries a weight, space, and cost and maintenance implication. With low cost UAVs there is generally not the space, weight allowance and practically unrealistic. The unique advantage with UAVs is that if stall occurs, and over land that is not populated, then there are no human injuries. Of course, there are risks to life if this happens over populated areas, e.g., when landing as most airports are in close approximation to people, and certainly the landing flight path. As a default, it has to be assumed that the UAV will be flying over populated areas and plans or designs to ensure safety is paramount.

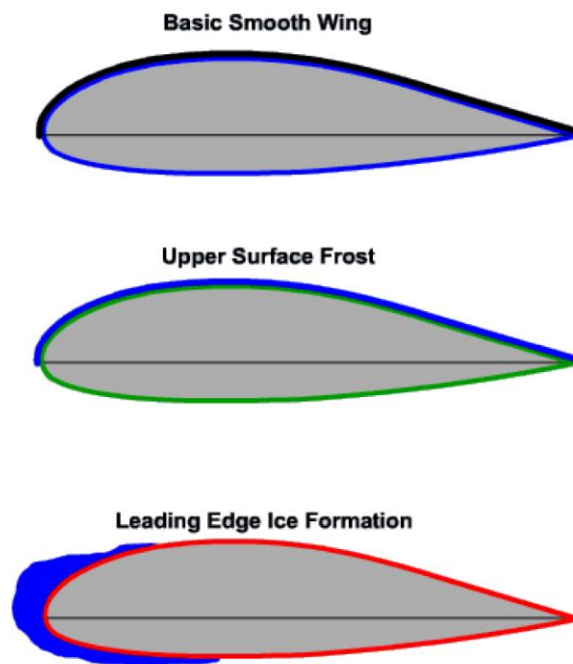


Figure 1: Aerofoils with and without ice formation

It is paramount that the influences of these are fully determined as the existing theory has not been applied in detail at low speeds and has been the preserve of high speed flight. In this research paper the influences of ice on lift and drag are investigated and presented.



2. Ice Formation

Below in figure 2 is a classical relationship of lift versus angle of attack for a smooth, leading edge ice and upper surface ice with upper surface ice the stall point decreases marginally and so does the maximum lift. With leading edge ice the stall point reduces but significantly and also the angle of attack when it starts. The implications can be significant. For example, taking off without knowing the ice is present means that lift will be less at take-off velocity and if the nose is pulled up too much stall will occur at a low altitude when recovery is not an option.

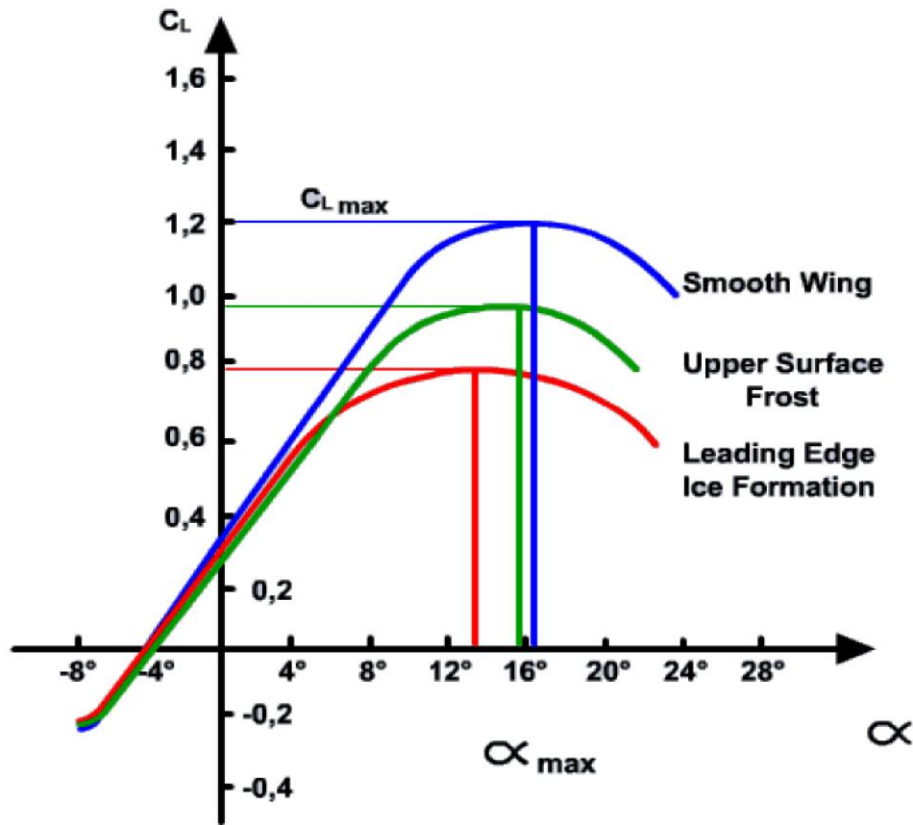


Figure 2: Classic lift v. angle of attack.

To summarise, ice formation reduces lift, adds weight, and increases drag. To maintain level flight extra thrust is needed or a pitch down to gain air speed; although that is only possible with altitude. When drag increases on low-speed aircraft the lift is always affected. Thus, without increased thrust altitude will be lost and when landing or taking-off that may result in stall and crash. This is more prominent for UAVs as remote piloting or auto-piloting has fewer options to correct error.



Figure 3: rubber leading edge ice removal mechanism.

Current research, Anderson 2005 *inter alia* has tended to focus on aspects where speed can be increased or minimum speeds determined for a design and when pre-flight procedures or in-flight heating can be used to eliminate build up at all stages of flight, (McAndrew IR, Godsey O and Navarro E., 2014). These are not options where smaller or very small unmanned vehicles can utilize the same methods. What has been lacking is the understanding at low speeds when this is the operational parameter of the vehicle. To this end a complete review and assessment of characteristics has been started with this research, (Dole & Lewis, 2000). Drag also changes as the angle of attack, α , increases. The influences of the drag with respect to the lift is addressed subsequently in this paper.

3. Literature of Ice Formation

As discussed previously ice forms due to many reasons and different stages of flight. Figure 3 is a classic example of where leading edge ice has built up whilst stationary. Note the ice that appears to be in stacks all joined together, (McAndrew & Moran, 2013). What is clear is that lift is drastically reduced and even when take-off speed achieved any attempt to pull the nose up will almost certainly result in stall and a crash given the low altitude as the stagnation point is low, (Bertins & Cummings, 2008)]. Thus minimal changes in pitch are possible. There are methods to remove the ice and prevent it from reoccurring and then take-off would be safe.



Figure 4: leading edge ice build-up on a wing.



Ice that is on a top surface of an aircraft wing reduces the lift and the angle of attack where separation of the air starts to occur, hence stall point. This is less than desirable but not as serious in comparison to leading edge ice formation, (McAndrew & Witcher, 2013). There are, however, different problems in preventing or removing ice and when it is dangerous to fly. Indeed, it is possible to fly safely under know parameters. Referring to figure 1, there is a loss of lift that can be overcome by increasing the speed at a slight angle of attack. Both reasonably acceptable for most of flight except take-off and landing, (Anderson, 2005). The nature of ice forming on the upper surface is, as shown, usually smooth enough to only slightly reduce lift in level flight, providing airspeed is not too slow, lift sufficient for level flight is possible. The consequences for drag are not considered until later in this research paper for any implication or direct aerodynamics.



Figure 5: upper surface ice in flight.

Of course, ice is a major problem all over the structure. Figure 6 below shows how ice forms instantly as small water drops come into contact with a cold surface and the energy is transferred following the second law of thermodynamics to instantly change from water to ice by fast cooling through the transition phase of ice and water. It is generally known as Super-cooled Large Droplet (SLD) conditions, similar to clear ice. Droplet size in this case is large and extends to unprotected parts. It forms ice significantly faster than normal icing conditions, (McAndrew, I. R., Witcher, K. & Navarro, E., 2015). This is more of a concern with lower altitude flying (historically that for unmanned vehicles). It is as applicable for any part of the structure and especially leading edges. This used to be a major concern for long ocean flights in the pioneering days of aviation, their only solution was to fly lower, although the increased density of air resulted in increased drag and fuel concerns and was seen as a major concern in the early days. Indeed, drag and low altitude have to be part of any low speed and low altitude flight design, (Fahlgren, 2011) Drag is addressed later in this research paper with discussions and also analysis.

Flights will always involve operating in environments where ice will be a problem and can never be removed. It can be managed and even allowed for in flight planning if the theory is fully understood, (McAndrew, I., Witcher, K., Navarro, E, 2016). Here the basic problems have been introduced and addressed. What and how evaluated form the remainder of this research paper and subsequently the implications. It will show unique responses for both lift and drag as the angle of attack changes from pitch down to pitch high and beyond separation.



Figure 6: nose ice formation.

In figure 7, below, it shows an aircraft being sprayed to prevent ice buildup before flight. This is time consuming, expensive and delays flights. Nevertheless, without this precaution there is a high risk of stall at take-off or when climbing to a cruise level. This becomes more critical when the temperatures drop, as temperature of -40°C are common at some international airports.



Figure 7: ice prevention before flight.

4. Methodology

Analysis of aerodynamics need to be assessed by accurate prediction and validated with experimentation for determine accurate descriptions explaining the theory. The first stage is to always determine sufficient detail for a review and extrapolations for future modelling. Here, initially, a smooth profile was used to determine the basic characteristics of lift and angle of attack, (Ferguson & Nelson, 2009). These parameters measured and the values of lift in Newton's, N, were measured for a datum wind speed. Using Reynolds numbers that ensure laminar air flow for this size of wing at a constant speed of 180 kmph assuming the wing has a constant cord length of 0.75m – typical for an unmanned vehicle. Ensuring the C_L and C_D obtained this was used to plot the corresponding graphs, shown below. The data from these two can be extrapolated to plot a Lilienthal Diagram (Polar Diagram) and this is shown in figure 11. A Lilienthal diagram can be used to determine the glide path for an aircraft based on the optimum lift and drag parameters.



Figure 8: wind tunnel measurements.

In table 1 below, the data is shown with various angles of attack, α , for three standard situations, smooth, leading edge ice and upper surface ice profiles. Ice buildup was simulated by using a rough sandpaper that was covering the leading edge or upper surface, this simple technique is accepted and changing the grade of paper can simulate various levels of ice, (McAndrew & Navarro, 2014). Here a thickness of approximately 5 mm of actual ice is represented on both leading edge and upper surface. Various other flight speeds and ice thickness were modelled, only one such set of results is represented in this research paper. Increments of 4° starting from -4° to 16° were set as datum points. Leading edge measurements were not readily obtainable with these values and they are shown separately at: $+4$, $+8$, $+9.5$ & $+11^\circ$ as the curve becomes steeper, more condensed and a sharper maximum and decline. These are nevertheless represented in the actual Lift v. angle of attack. A combination of both leading edge and upper surface has not been included, it is heavily weighted by leading edge characteristics and follows that pattern. For now, a direct measurement is intended to make comparisons with the typical theory in figure 2.

Table1:Lift v. Angle of Attack for the three principal profiles of smooth, upper and leading edge ice formation.

Angle of attack, in degrees	-4	0	+4	+8	+12	+16
Lift (smooth), N	-3	8	21	33	37	30
Lift (upper surface), N	-6	5	16	26	30	26
Angle of attack, in degrees			+4	+8	+9.5	+11
Lift (leading edge), N			11	21	24	22

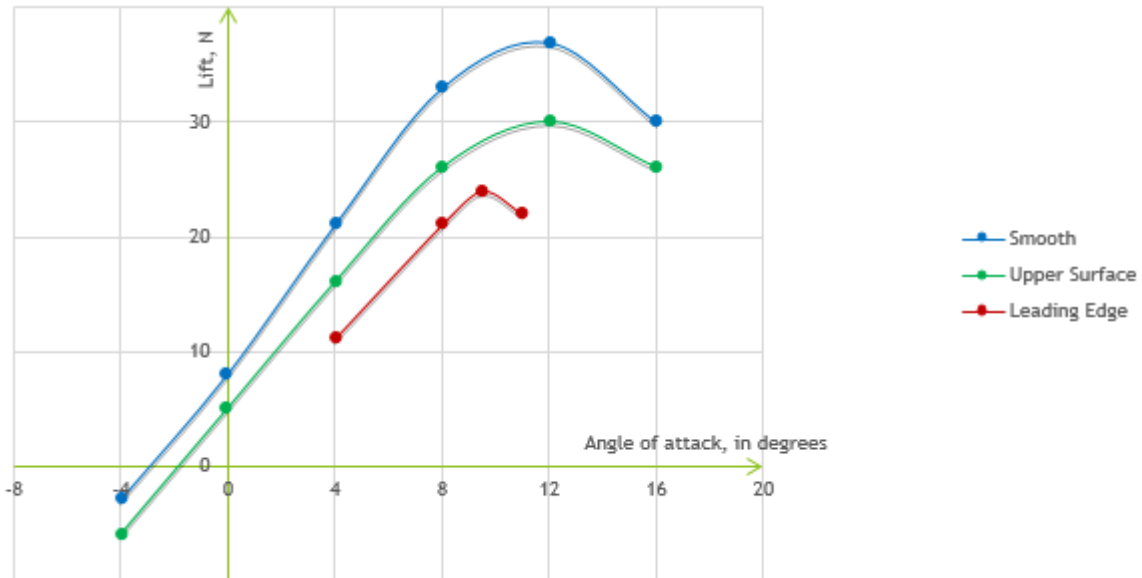


Figure 9: lift v angle of attack.

This graph follows the principles introduced in figure 2. It does, however, represent a unique situation of the parameters. First, considering the smooth surface represented by the blue line. For this profile of aircraft wing a 4° spread from start of separation (maximum point) to the final point where no lift is generated and stall is total. Upper surface lift shows a significant reduction in lift, approximately 20% at the maximum value. Where this differs from conventional theory is that the maximum value is reduced but occurs at the same angle of attack, it cannot be determined if this is a typical or unique from this wings profile. The point at which first separation occurs to stall (maximum to last point) is the same number of degrees as for a smooth ice free wing profile. This is very dependent upon the profile and does not represent an established fact. There may be parallel or separate reason, not investigated here and only reported. Thirdly, the curve for the leading edge identifies several unique features of interest. This also reduces the amount of lift generated and that can at certain times of flight very important. Thus, at its maximum value of approximately 33% or about one-third. Its maximum point is almost 3° lower than for a smooth or upper surface ice. More significantly is that an angle of attack from maximum (start of separation to stall) is halved from 4 to 2 degrees. This would mean very little time for a pilot to respond to a first stall warning, coupled with a remote pilot and the time delay would make it inevitable that stall recovery is needed to gain control.

Drag also changes when the angle of attacks is changed and considerably increases as the angle of attack goes beyond 8° . With an increased angle of attack the front profile cross-section of area also increases, thus more energy is needed to move forward (drag). Higher values further increase the drag and even past the separation point this increases as turbulence flow further compounds the drag. This is true for any smooth profile or combination of ice build-up. What is interesting is the effects of ice on the leading edge and upper surface. Table 2 represents the results of measured drag from the wind tunnel experiments shown in figure 9. The section used was a uniform and continuous profile that ignores wing tip influences.

The data shown in table II also extends from -4° to 16° to directly compare with the lift data represented in figure 6. Drag values where the leading edge is fully iced the values only ranges from -4° to $+8^\circ$ and all drag forces are measured in Newton's were beyond the calibrated range of measurements.



Table 2: Drag v. Angle of Attack for the three principal profiles

Angle of attack, in degrees	-4	0	+4	+8	+12	+16
Drag (smooth), N	5	2	3.5	6.5	11	17
Drag (upper surface), N	6	5	8	13	17.5	24

Angle of attack, in degrees	-4	0	+4	+8
Drag (leading edge), N	7.5	8	14.5	24

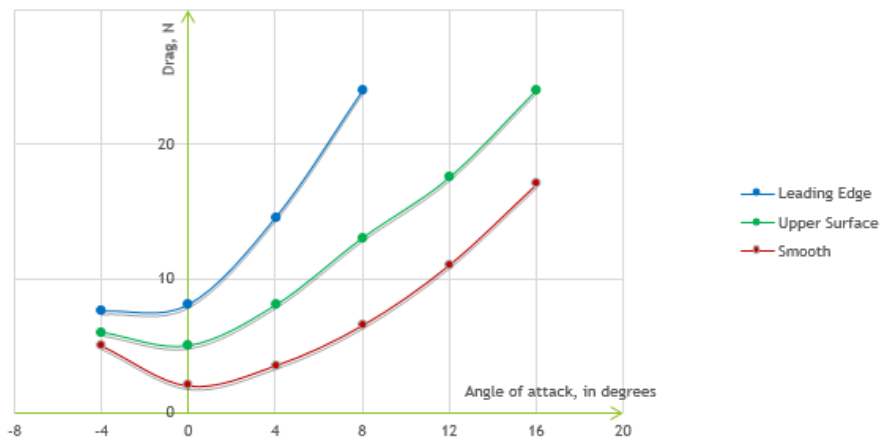


Figure 10: drag v. angle of attack.

Table 3: Lilienthal Diagram data from Drag v. Lift at a given Angle of Attack for the three principal profiles of

Angle of attack Smooth	Drag	Lift	Angle of attack Upper	Drag	Lift	Angle of attack Leading	Drag	Lift
-4	5	-3	-4	6	-6	4	7.5	11
0	2	8	0	5	5	8	8	21
4	3.5	21	4	8	16	9.5	14.5	24
8	6.5	33	8	13	26	11	24	22
12	11	37	12	17.5	30			
16	17	30	16	24	26			

Table 3 shows the data from tables 1 & 2 for determining the plot values in figure 11. Key angle of attacks are shown where they form the tangent line, hence, glide path for optimum descent.

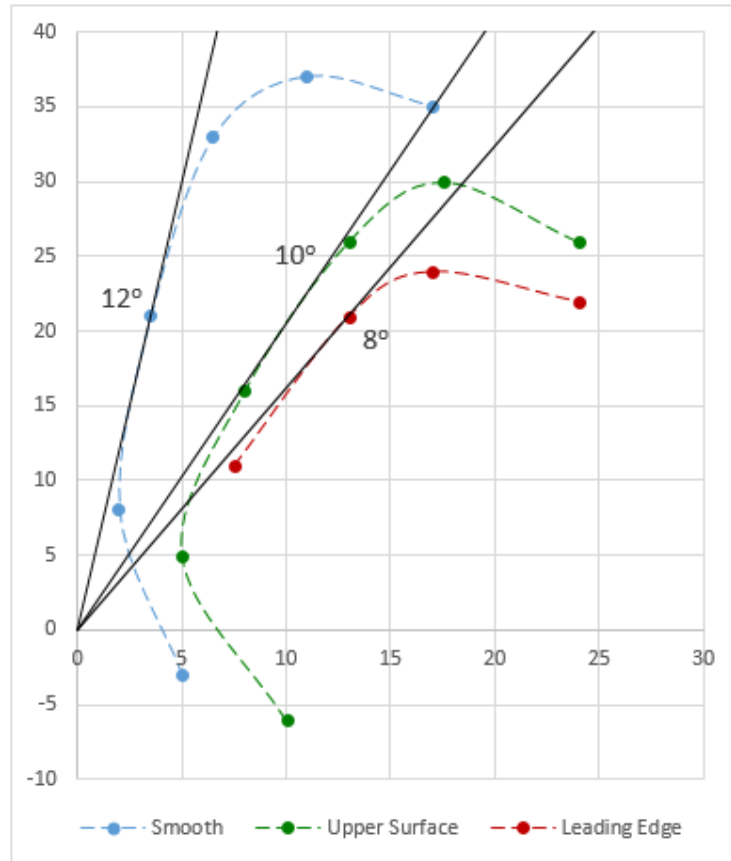


Figure 11: Lilienthal Diagram for the three wing configurations.

5. Discussion and Implications

The lift and drag for the smooth profile are shown and that the characteristic curve profiles follows that expected and can be further developed to determine the corresponding Polar Diagram, Lilienthal. A maximum drag is shown at 16° as this corresponds the stall value for this particular wing profile. When iced on the upper surface is measured the increase in drag can be clearly seen to be almost double at the lower angles of attack and approximately 50% higher at 16° . When the leading edge is iced the drag substantially increases immediately. At a zero angle of attack it is higher than -4° , which is different to the other two plots that drop first before again increasing. At 8° the drag is higher than the upper surface ice at 16° ; and 50% higher than smooth at 16° .

6. Analysis of Results

Any ice on the upper or leading edge is a problem whilst considering aerodynamic lift and drag at any speed, although at lower speeds a greater concern. On commercial aircraft before take-off there are ice removal and prevention methods to ensure a smooth surface and thereby ensuring the lift, pitch up and control are as expected. In flight with ice buildup the aircraft can descend to an altitude where the ice will melt, traditional on older aircraft. Modern aircraft have heat mats, re-circulation of heated air from the engines and other methods for both ice removal and prevention. On smaller unmanned vehicles the space and weight restrictions



do not afford this luxury and when flying the only option is to descend and wait until the ice melts as the higher temperature influence the ice, (Simmons, 2009). Commercial aircraft also can ensure ice is prevented prior to landing, unmanned vehicles cannot emulate. Where unmanned are used in hostile regions (war zones) lower altitudes will make it vulnerable to attack from ground attack.

Landing an unmanned vehicle with ice on the upper surface will seriously limit the pitch control as this is drastically limited, over pitch from these results but still within the classical envelope will result in stall, (Moir & Seabridge, 2012). With ice on the leading edge the situation is worse. Stall occurs at only 8° of pitch, and the drag that will slow down the surface speed further compound the problem as with lower speed stall occurs earlier.

The landing with ice, whether leading edge or upper surface, will have the most significant effect on landing. Take-off can be delayed and ice prevention spray used. In flight the aircraft can descend to an altitude that has a higher outside surface temperature. Landing needs pitch control and maintained surface ground speed. Here, pitch movement is limited, even the movement for any particular aircraft can be achieved as it is for rudder movement in-flight. Speed then becomes the significant, and needs addressing.

Landing usually has minimal fuel and payload, thus the thrust generated by the engine is useable. Measuring these known parameters for any profile at a range of speed and the lift and drag can be determined. These can be incorporated into the operational envelope of flight and without the ability to remove ice, leading edge or upper surface, can be accommodated.

Adding to the findings are the fact of the Lilienthal Diagram. This clearly shows that for a smooth surface the gradient tangent point at 12° is the optimum glide path ratio for landing, which matches decent with drag and including lift available. With the upper surface ice the glide path angle is now reduced to 10° and this means a longer landing cycle that is not desirable. Trying to land at a steeper angle can increase flight instability with excess drag, a higher fuel needed. When the leading edge ice is modelled then the values change even more with greater implications. Now, the glide slope will be 8° and would require a very long landing path. Alternatively, if the situation is considered practically the reverse can be considered. If, for any reason, ice is on a UAV that does not mean a dangerous landing, just one that means the approach has to be increased greatly. The difference in height from 20 miles with a 4° difference will be about 1.4 miles in altitude. A flatter approach that is kept at 8° does suggest that control can be achieved and a safe landing.

7. Conclusion

In conclusion for this research it has shown theory does not match practice in this case. A standard wing section had been used for three typical cases, smooth, upper surface ice and leading edge ice. Findings from the wind tunnel identify the theory for general wing design do not match sufficiently closely to be reliable. In this a small wing similar to those used with unmanned aerial vehicles it showed that the point of first separation to stall in both magnitude and position is different. Drag, as the lift changes also has its own unique responses where the effects have far more reaching consequences. These conditions may be disastrous for flight at take-off and landing. Taking these further and determining the glide path shows how with ice on either the leading edge or upper surface will drastically alter the optimum approach for minimizing drag, balancing lift and being fuel efficient. The sensitivity of the glide path is high for the upper wing ice and leading edge and would need flight stability to ensure no stall.

Future research will use this data and measure a wider range of air speeds to determine all lifts and drag for smooth, leading edge and upper surfaces. In addition various wing profiles used with UAVs will be investigated. These will be used to expand on the basic details and determine more Polar Diagrams and establish more detail about landing parameters for many glide paths.



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