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Natural Convection from Circular Cylinders

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Sandra K. S. Boetcher

Natural Convection from Circular Cylinders



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Sandra K. S. Boetcher

Natural Convection from Circular Cylinders

 Springer

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Preface

A few years ago, I was doing research on thermal plumes emanating from vertical cylinders. In order to validate the numerical model, I wanted to compare the Nusselt number of the cylinder from the simulation to an experimental correlation. I thought finding an experimental Nusselt number correlation for natural convection from vertical cylinders would be a trivial task. Much to my surprise, after consulting many well-known heat transfer textbooks, I found out that this was not the case. Instead, the textbooks presented the criteria for treating the vertical cylinder as a vertical plate, which is valid in cases where the boundary-layer thickness is small compared to the diameter of the cylinder. Unfortunately, if one cannot make this assumption (as it was in my case), they are directed by the authors of the textbooks to some hard-to-find literature. Thus began my journey to search for and compile natural convection Nusselt number correlations from cylinders. This book attempts to be a single, updated source on the subject.

Daytona Beach, FL, May 2014

Sandra K. S. Boetcher

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Nomenclature

A	Coefficient
a	Constant or exponent
B	Coefficient
b	Constant
C	Constant or exponent
C_L	Function
c	Constant
c_p	Specific heat capacity [J/kg-K]
D	Diameter of the cylinder [m]
d	Constant
E	Function
e	Constant
g	Gravitational constant [m/s ²]
Gr_D	Diameter-based Grashof number, $g\beta(T_{cylinder}-T_\infty)D^3/\nu^2$
Gr_D^*	Modified diameter-based Grashof number, $g\beta qD^4/(k\nu^2)$
Gr_L	Height-based Grashof number, $g\beta(T_{cylinder}-T_\infty)L^3/\nu^2$
Gr_L^*	Modified height-based Grashof number, $g\beta qL^4/(k\nu^2)$
Gr_r^*	Modified radius-based Grashof number, $g\beta q r^4/(k\nu^2)$
Gr_x	Local Grashof number, $g\beta(T_{cylinder}-T_\infty)x^3/\nu^2$
h	Local heat transfer coefficient [W/m ² -K]
\bar{h}	Average heat transfer coefficient [W/m ² -K]
k	Thermal conductivity [W/m-K]
L	Height of the cylinder [m]
M	Constant
m	Constant, exponent, or function
N	Constant
n	Exponent or function
Nu_D	Diameter-based average Nusselt number, $\bar{h}D/k$
Nu_L	Height-based average Nusselt number, $\bar{h}L/k$
$Nu_{L,fp}$	Height-based average Nusselt number of flat plate

Nu_x	Local Nusselt number, hx/k
$Nu_{x,fp}$	Local Nusselt number of flat plate
Nu_θ	Local Nusselt number of horizontal cylinder hD/k
p	Function
Pr	Prandtl number, $c_p\mu/k$
q	Heat flux [W/m^2]
R	Function
r	Radius [m]
Ra_D	Diameter-based Rayleigh number, $Gr_D Pr$
Ra_D^*	Modified diameter-based Rayleigh number, $Gr_D^* Pr$
Ra_L	Height-based Rayleigh number, $Gr_L Pr$
Ra_x^*	Local modified Rayleigh number, $Gr_x^* Pr$
T	Temperature [K]
$T_{cylinder}$	Temperature at the surface of the cylinder [K]
T_e	Reference temperature [K]
T_m	Average temperature [K]
T_∞	Temperature of the ambient environment [K]
X	Function
x	Function or coordinate [m]

Greek

β	Isobaric coefficient of thermal expansion [K^{-1}]
δ	Boundary layer thickness [m]
θ	Angle of inclination from the vertical (inclined cylinders) or position on horizontal cylinder [degrees]
μ	Dynamic viscosity of the fluid [Pa-s]
ν	Kinematic viscosity of the fluid [m^2/s]
ξ	Curvature parameter, $(4L/D)(Gr_L/4)^{-1/4}$ or $(4x/D)(Gr_x/4)^{-1/4}$
φ	Function
ϕ	Angle of inclination from the horizontal (inclined cylinders) or bisecting angle (horizontal cylinders)

Chapter 1

Introduction

Abstract Natural convection heat transfer from circular cylinders in quiescent environments is a subject which has been studied extensively. Although a well-established topic, to the best knowledge of the author no single source of reference is found for natural convection from circular cylinders for varying orientations (horizontal, vertical, and inclined). This book attempts to be a reference for Nusselt number correlations for circular cylinders of all inclinations including horizontal and vertical.

Keywords Natural convection · Vertical cylinder · Horizontal cylinder · Inclined cylinder · Experimental · Computational · Analytical · Nusselt numbers

Natural convection heat transfer from circular cylinders in quiescent environments is a subject which has been studied extensively. Although a well-established topic, to the best knowledge of the author, no single source of reference is found for natural convection from circular cylinders for varying orientations (horizontal, vertical, and inclined). Moreover, the last major reference often cited is the review paper of Morgan [2], which was published in 1975. This book attempts to be a one-stop resource on the topic.

The most amount of literature exists for horizontal cylinders. Studying the natural convection flow around horizontal cylinders is important for many practical engineering applications as well as benchmarking numerical simulations. At small Rayleigh numbers, the horizontal cylinder behaves like a line heat source. For larger Rayleigh numbers, a boundary layer forms around the cylinder that must be considered. Although many investigations on the subject exist, there is a wide dispersion in analytical, numerical, and experimental data. The dispersion exists in analytical and numerical methods because of using the simplified boundary-layer equations, boundary condition assumptions, and solution domain size. Natural convection experiments are notoriously difficult to perform because it is very hard to isolate the natural convection phenomena from other influencing factors such as heat conduction losses, measurement issues, radiation effects, size of experimental chambers, and external temperature and velocity interferences.

Natural convection heat transfer from circular vertical cylinders is a topic which has also been studied extensively. Although a large amount of literature exists on

the topic, it is difficult to find correlations in many standard heat transfer textbooks. For the most part, the texts display a criterion for approximating the natural convection heat transfer from a vertical cylinder by using the correlations for a vertical flat plate. This is acceptable if the boundary layer thickness of the vertical cylinder is small compared to the diameter of the cylinder. However, if this is not the case, the reader is directed to hard-to-find literature references.

Like horizontal cylinders, there are many discrepancies in the available data. This is again due to the fact that natural convection experiments are difficult. Also, for analytical and numerical techniques, the boundary layer approximation is often used (all pressure gradients are zero, and streamwise second derivatives are neglected) and the boundary layer thickness at the leading edge is assumed to be zero. The orientation of the cylinder with respect to a surface is also an issue. Many investigations assume the cylinder is floating in space, whereas more realistically, the cylinder is attached to a surface. Lastly, the conditions at the ends of the cylinder, whether they are heated or not, also sometimes cause disparity.

Finally, but not surprisingly, the least amount of work has been done for natural convection heat transfer from round inclined cylinders. The main reason for this is that the natural convection heat transfer from horizontal and vertical cylinders is a two-dimensional problem, and the natural convection from inclined cylinders is a more complex three-dimensional problem. In fact, prior to 1957, no literature on the subject could be found according to Farber and Rennat [1].

References

1. Farber E, Rennat H (1957) Variation of heat transfer coefficient with length-inclined tubes in still air. *Ind Eng Chem* 49:437–440
2. Morgan V (1975) The overall convective heat transfer from smooth circular cylinders. *Adv Heat Transf* 11:199–264

Chapter 2

Natural Convection Heat Transfer From Horizontal Cylinders

Abstract A vast amount of literature exists for natural convection from horizontal cylinders. This chapter separates the field into two main time periods: early investigators and modern developments. Early investigators focused on analytical and experimental techniques, and modern investigations have focused on using computational fluid dynamics. Nusselt number correlations are presented from many sources for natural convection from horizontal cylinders.

Keywords Natural convection · Horizontal cylinder · Experimental · Computational · Analytical · Nusselt numbers

2.1 Introduction

Heat transfer from horizontal cylinders has the most amount of literature out of all of the orientations. This is likely due to the symmetric, two-dimensional nature of horizontal cylinders. According to Morgan [51], who in 1975 published an all-encompassing review article on the natural convection from smooth, circular cylinders, there is a wide dispersion in experimental results due to axial heat conduction losses to the supporting structures of the horizontal cylinders, temperature measurement location, interference of the temperature and velocity fields by convective fluid movements, and the utilization of small containing chambers for the experiments. Champagne et al. [7] showed that the temperature is uniform over at least the center third of a heated cylinder if $L/D > 200$, but many early experimental investigators used cylinders having $L/D < 10$.

At small Rayleigh numbers, the heat transfer from a horizontal cylinder behaves like a line heat source. For larger Rayleigh numbers, i.e., $10^4 \leq Ra_D \leq 10^8$, the flow forms a laminar boundary layer around the cylinder [39]. At even higher Rayleigh numbers, it is expected that the flow becomes turbulent.

2.2 Temperature Boundary Conditions

2.2.1 Early Investigators

One of the earliest studies of natural convection from horizontal cylinders was that of Ayrton and Kilgour in 1892 [4]. In the manuscript, the authors looked at the thermal emission of thin, long horizontal wires with $0.031 \text{ mm} \leq D \leq 0.356 \text{ mm}$ and $790 \leq L/D \leq 9000$ in air. The heat loss of the wires was calculated by multiplying the current by the potential difference in the wire. Morgan [51] correlated the experimental results into the following equation.

$$\text{Nu}_D = 1.61(\text{Gr}_D\text{Pr})^{0.141} \quad (2.1)$$

for $10^{-4} \leq \text{Gr}_D\text{Pr} \leq 3 \times 10^{-2}$.

In 1898 and 1901, Petavel [56, 57] performed experiments on a thin wire with a diameter of 1.1 mm and aspect ratio of $L/D = 403$ in air. Morgan [51] correlated the data

$$\text{Nu}_D = 1.05(\text{Gr}_D\text{Pr})^{0.14} \quad (2.2)$$

for $0.1 \leq \text{Gr}_D\text{Pr} \leq 3 \times 10^2$, and

$$\text{Nu}_D = 0.562(\text{Gr}_D\text{Pr})^{0.25} \quad (2.3)$$

for $3 \times 10^2 \leq \text{Gr}_D\text{Pr} \leq 2 \times 10^5$.

Kennelly et al. [35] studied the free convection from small copper wires in air. The diameter of the wire studied was 26.2 mm, and the aspect ratio was 4540. The correlation of data [51] from that paper results in the following for $10^{-2} \leq \text{Gr}_D\text{Pr} \leq 0.3$.

$$\text{Nu}_D = 0.945(\text{Gr}_D\text{Pr})^{0.118} \quad (2.4)$$

In 1911, Wamsler [69] experimentally determined natural convection from a horizontal thin cylinder in air for $20.5 \text{ mm} \leq D \leq 89 \text{ mm}$ and $34 \leq L/D \leq 147$. The data resulted in the following Nusselt number correlation [51]

$$\text{Nu}_D = 0.480(\text{Gr}_D\text{Pr})^{0.25} \quad (2.5)$$

for $3 \times 10^4 \leq \text{Gr}_D\text{Pr} \leq 3.5 \times 10^6$.

Langmuir [42] performed experiments on horizontal platinum wires in air with diameters between 0.004 cm and 0.0510 cm. Morgan [51] correlated the results

$$\text{Nu}_D = 0.81(\text{Gr}_D\text{Pr})^{0.065} \quad (2.6)$$

for $4.5 \times 10^{-5} \leq \text{Gr}_D\text{Pr} \leq 10^{-2}$, and

$$\text{Nu}_D = 1.12(\text{Gr}_D\text{Pr})^{0.125} \quad (2.7)$$

for $10^{-2} \leq \text{Gr}_D\text{Pr} \leq 0.6$.

In 1922, Davis [14] performed natural convection experiments on thin horizontal wires ($D = 0.0083$ and 0.0155 cm) in several fluids: air, CCl_4 , aniline, olive oil, and glycerin. The following correlation for $10^{-4} \leq \text{Gr}_D\text{Pr} \leq 10^6$ is

$$\text{Nu}_D = 0.47(\text{Gr}_D\text{Pr})^{0.25} \quad (2.8)$$

In 1924, Rice [61] experimentally determined the Nusselt numbers from horizontal cylinders in air. The data correlate into the following equation

$$\text{Nu}_D = \frac{2}{\ln(1 + 2/0.47(\text{Gr}_D\text{Pr})^{1/4})} \quad (2.9)$$

for $10^{-2} \leq \text{Gr}_D\text{Pr} \leq 10^4$. And in 1923 [60], Rice experimentally determined (and Morgan [51] correlated) that

$$\text{Nu}_D = 0.97(\text{Gr}_D\text{Pr})^{0.203} \quad (2.10)$$

for $4 \times 10^3 \leq \text{Gr}_D\text{Pr} \leq 6 \times 10^6$.

Also in 1924, Nelson [53] studied the natural convection of hot wires in various liquids. The experimental study was that of a horizontal cylinder that was 0.033 cm in diameter and 16.9 cm long. Heat loss results were obtained for water and alcohol for $1.4 \leq \text{Gr}_D\text{Pr} \leq 66$ and correlated by Morgan [51]

$$\text{Nu}_D = 1.32(\text{Gr}_D\text{Pr})^{0.102} \quad (2.11)$$

Koch [37] performed experiments in air in 1927. He used cylinders with aspect ratios between 20 and 152. The correlated results [51] are as follows:

For $4 \times 10^3 \leq \text{Gr}_D\text{Pr} \leq 4 \times 10^5$

$$\text{Nu}_D = 0.412(\text{Gr}_D\text{Pr})^{0.25} \quad (2.12)$$

and for $4 \times 10^5 \leq \text{Gr}_D\text{Pr} \leq 6 \times 10^6$

$$\text{Nu}_D = 0.286(\text{Gr}_D\text{Pr})^{0.28} \quad (2.13)$$

In 1929, Nusselt [54] analytically determined heat transfer coefficients for horizontal cylinders in air or liquids for $10^4 \leq \text{Gr}_D\text{Pr} \leq 10^8$.

$$\text{Nu}_D = 0.502(\text{Gr}_D\text{Pr})^{0.25} \quad (2.14)$$

Schurig and Frick [64], in 1930, experimentally determined average Nusselt numbers for horizontal bare conductors in air for $45 \leq L/D \leq 286$ and $2.7 \times 10^3 \leq Gr_D Pr \leq 8.2 \times 10^5$. Their findings were correlated by Morgan [51] and appear below.

$$Nu_D = 0.57(Gr_D Pr)^{0.24} \quad (2.15)$$

In 1932, Ackermann [1] experimentally determined natural convection heat loss from a horizontal cylinder with an aspect ratio of 2.8 in water for $10^7 \leq Gr_D Pr \leq 4.5 \times 10^8$. The following correlation [51] resulted.

$$Nu_D = 0.14(Gr_D Pr)^{0.32} \quad (2.16)$$

Also in 1932, King [36] analytically determined Nusselt numbers for horizontal cylinders in air and/or liquids.

For $10^3 \leq Gr_D Pr \leq 10^6$

$$Nu_D = 0.53(Gr_D Pr)^{0.25} \quad (2.17)$$

and for $10^6 \leq Gr_D Pr \leq 10^{12}$

$$Nu_D = 0.13(Gr_D Pr)^{0.33} \quad (2.18)$$

In 1933, Jodlbauer [33] performed experiments in air with $55 \leq AR \leq 140$ and $3.9 \times 10^4 \leq Gr_D Pr \leq 3.6 \times 10^6$. Morgan [51] correlated the results

$$Nu_D = 0.480(Gr_D Pr)^{0.25} \quad (2.19)$$

Jakob and Linke [32], in 1935, performed experiments in liquids for an aspect ratio of 4.3.

For $10^4 \leq Gr_D Pr \leq 10^8$

$$Nu_D = 0.555(Gr_D Pr)^{0.25} \quad (2.20)$$

and for $10^8 \leq Gr_D Pr \leq 10^{12}$

$$Nu_D = 0.129(Gr_D Pr)^{0.333} \quad (2.21)$$

In 1936, Hermann [31] analytically determined the average Nusselt numbers for horizontal cylinders in gas environments. The following equation is valid for $10^4 \leq Gr_D Pr \leq 5 \times 10^8$.

$$Nu_D = 0.424(Gr_D Pr)^{0.25} \quad (2.22)$$

In 1938, Martinelli and Boelter [47] (along with a similar study in 1940 [48]) studied the effect of vibration on the heat transfer by natural convection from a horizontal cylinder. The investigators placed a heated cylinder that was 19.05 mm (0.75 in.) and

320.675 mm (12.625 in.) long. The investigators found a critical Reynolds number, based on the angular velocity of the vibrations and the ratio of the displacement amplitude to the tube diameter, for which the effects of the vibrations increased the rate of heat transfer. The authors caution the reader about the results because the work revealed that the diameter–displacement amplitude ratio was a descriptive variable; however, the effect of this variable was not discernible in the experiments.

In 1942, Lander [41] presented Nusselt number curves for natural convection from horizontal cylinders for gases and liquids. The following correlation is for $10^3 \leq Gr_D Pr \leq 10^7$

$$Nu_D = 0.49(Gr_D Pr)^{0.25} \quad (2.23)$$

and for $10^8 \leq Gr_D Pr \leq 10^9$

$$Nu_D = 0.12(Gr_D Pr)^{0.33} \quad (2.24)$$

In 1948, Elenbaas [18] determined that for horizontal cylinders

$$Nu_D \exp(-6/Nu_D) = \frac{Gr_D Pr}{235 f(Gr_D Pr)} \quad (2.25)$$

where $f(Gr_D Pr)$ has been determined experimentally. For $Gr_D Pr < 10^4$, $f(Gr_D Pr) = 1$.

Senfleben [65] in 1951 for air, gases, and liquids determined the average Nusselt numbers for horizontal cylinders. For $10^5 \leq Gr_D Pr \leq 10^8$

$$Nu_D = \frac{2}{X} \left[1 - \frac{0.033}{X(Gr_D Pr)^{0.25}} \left\{ \left(1 + \frac{X(Gr_D Pr)^{0.25}}{0.033} \right)^{0.5} - 1 \right\} \right] \quad (2.26)$$

where

$$X = \ln \left[1 + \frac{4.5}{(Gr_D Pr)^{0.25}} \right] \quad (2.27)$$

and for *large* values of $Gr_D Pr$

$$Nu_D = 0.41(Gr_D Pr)^{0.25} \quad (2.28)$$

In 1953, Kyte et al. [40] investigated the effect of reduced pressure (0.1 mmHg to atmospheric) on the natural convection of horizontal cylinders. At reduced pressures, the thickness of the boundary layer becomes large, the gas becomes rarefied, and free-molecule conduction becomes important. Kyte et al. experimentally determined the

average Nusselt numbers for a cylinder of diameter 0.078 mm and aspect ratio of 1910. For $10^{-7} \leq Gr_D Pr \leq 10^{1.5}$

$$Nu_D = \frac{2}{\ln [1 + 7.09/(Gr_D Pr)^{0.37}]} \quad (2.29)$$

and for $10^{1.5} \leq Gr_D Pr \leq 10^9$

$$Nu_D = \frac{2}{\ln [1 + 5.01/(Gr_D Pr)^{0.26}]} \quad (2.30)$$

In 1954, Collis and Willams [13] performed experiments on horizontal platinum wires with $0.0003 \text{ cm} \leq D \leq 0.0041 \text{ cm}$ in air. In order to establish negligible end effects, all wires had an aspect ratio $L/D \geq 20,000$. For $10^{-10} \leq Gr_D Pr \leq 10^{-3}$, Morgan [51] correlated the results

$$Nu_D = 0.675(Gr_D Pr)^{0.058} \quad (2.31)$$

Etemad [19], in 1955, performed natural convection experiments on rotating horizontal cylinders in air. The cylinders had diameters of 60.4 and 63.5 mm, and aspect ratios of 7.1 and 7.5, respectively. Etemad tested both rotating cylinders and stationary cylinders. For the stationary cylinders, Etemad found that for $1.2 \times 10^5 \leq Gr_D Pr \leq 1.3 \times 10^6$, the average Nusselt number is

$$Nu_D = 0.456(Gr_D Pr)^{0.25} \quad (2.32)$$

In 1955, another investigator who studied the effect of vibration on natural convection heat transfer was Lemlich [43]. Lemlich experimentally studied heated wires with a diameter of 1.01 mm and aspect ratio of 917 in air. For the case with no vibration and $6 \times 10^2 \leq Gr_D Pr \leq 6 \times 10^3$, Morgan [51] correlated the experimental data into the following equation

$$Nu_D = 0.45(Gr_D Pr)^{0.22} \quad (2.33)$$

In 1956, van Der Hegge Zijnen [68] combined correlation equations found in the literature into a single correlation equation with a wider range of applicability. He reviewed the current literature and took the empirical correlations of Rice [61] (Eq. 2.9), Elenbaas [18] (Eq. 2.25), and Senftleben [65] (Eq. 2.26) and combined them into a single correlation shown below.

$$Nu_D = 0.35 + 0.25(Gr_D Pr)^{1/8} + 0.45(Gr_D Pr)^{1/4} \quad (2.34)$$

For low Grashof number, in 1956, Fischer and Dosch [25] experimentally determined average Nusselt numbers in air for horizontal cylinders with diameters of

0.0022–0.114 mm and aspect ratios between 807 and 12800. For $3 \times 10^{-5} \leq Gr_D Pr \leq 8 \times 10^{-3}$. Morgan [51] developed the following correlation

$$Nu_D = 0.862(Gr_D Pr)^{0.0678} \quad (2.35)$$

In 1956, Beckers et al. [6] performed natural convection experiments on very thin horizontal platinum wires (0.025–0.66 mm) in air, alcohol, paraffin oil, and water at very low Grashof numbers $10^{-8} \leq Gr_D \leq 1$. They concluded that at very low Grashof numbers, the Nusselt number is not dependent on the Prandtl number. The correlation given in [6] is

$$Nu_D = 0.95 Gr_D^{0.08} \quad (2.36)$$

In 1958, Kays and Bjorklund [34] experimentally determined heat transfer coefficients for rotating horizontal cylinders. As part of their study, they tested nonrotating cylinders. Their experimental data verified the correlations presented by Etemad [19], Eq. (2.32), and McAdams [49].

In 1960, Tsubouchi and Sato [66] experimentally determined natural convection coefficients for horizontal wires in air. The range of diameters used in the study was from 0.00489 to 0.061 mm and anywhere from 40 to 150 mm long depending on the wire diameter. For $10^{-8} \leq Gr_D \leq 10^{-1}$

$$Nu_D = 0.812 Gr_D^{1/15} \quad (2.37)$$

In 1961, Fand and Kaye [20] investigated the effect of acoustic vibrations on the natural convection from horizontal cylinders in air. The test specimen was a cylinder that was 19 mm in diameter and 159 mm long. For a baseline case, Fand and Kaye determined the heat transfer from the cylinder without sound. These data were correlated by Morgan [51] and presented here for $10^4 \leq Gr_D Pr \leq 4 \times 10^4$

$$Nu_D = 0.485(Gr_D Pr)^{0.25} \quad (2.38)$$

Rebrov [59], in 1961, performed natural convection experiments on horizontal cylinders with $1.31 \text{ mm} \leq D \leq 9.9 \text{ mm}$ and $61 \leq L/D \leq 458$ in rarefied air for $10^{-7} \leq Gr_D Pr \leq 4 \times 10^8$. The experimental results were correlated into the following equation

$$Nu_D = \left[0.98 - 0.01(\log Gr_D Pr)^2 \right] (Gr_D Pr)^x \quad (2.39)$$

where x is

$$x = 0.14 + 0.015 \log(Gr_D Pr) \quad (2.40)$$

Also in 1961, Zhukauskas et al. [73] studied the effect of ultrasonic waves on the heat transfer of bodies in water and oils. The diameter of the horizontal cylinder was 8 mm, and the aspect ratio was approximately 20. For the baseline case without ultrasound, the following average Nusselt number expression was obtained.

$$\text{Nu}_D = 0.50(\text{Gr}_D\text{Pr})^{0.25} \quad (2.41)$$

The preceding equation is good for $1.5 \times 10^4 \leq \text{Gr}_D\text{Pr} \leq 2.5 \times 10^6$.

In 1962, Deaver et al. [15] experimentally investigated the effect of oscillations on a horizontal wire of 0.178 mm diameter and approximately 2 m long in water. For the case without the vibrations, the average Nusselt number was found to be

$$\text{Nu}_D = 1.15(\text{Gr}_D\text{Pr})^{0.15} \quad (2.42)$$

for $0.2 \leq \text{Gr}_D\text{Pr} \leq 20$.

In 1963, Fand and Kaye [21] studied the effect of vertical vibrations on the heat transfer of a 22.2-mm-diameter cylinder in air with an aspect ratio of approximately 25. Morgan [51] correlated the data for the control case (no vibration) which resulted in the following equation

$$\text{Nu}_D = 0.495(\text{Gr}_D\text{Pr})^{0.25} \quad (2.43)$$

for $2 \times 10^4 \leq \text{Gr}_D\text{Pr} \leq 6 \times 10^4$.

Another study on the effect of vibration on the natural convection from horizontal cylinders was done by Lemlich and Rao [44] in 1965. Lemlich and Rao tested a cylinder with $D = 1.25$ mm and $L = 490$ mm in water and glycerin. For the case with no vibration, Morgan [51] correlated the data. For $1.8 \times 10^2 \leq \text{Gr}_D\text{Pr} \leq 1.9 \times 10^3$

$$\text{Nu}_D = 0.58(\text{Gr}_D\text{Pr})^{0.25} \quad (2.44)$$

In 1966, Tsubouchi and Masuda [67] performed natural convection experiments in air on horizontal cylinders which were 21.5 mm in diameter and 200 mm in length. The purpose of the study was to compare cylinders with grooved surfaces with cylinders of smooth surfaces. For the smooth-surfaced cylinders, Tsubouchi and Masuda developed the following formula for $2.3 \times 10^4 \leq \text{Gr}_D \leq 7.5 \times 10^4$

$$\text{Nu}_D = 0.44 \text{Gr}_D^{0.25} \quad (2.45)$$

Also in 1966, Penney and Jefferson [55] performed experiments to investigate the heat transfer from an oscillating horizontal wire to water and ethylene glycol. The diameter of the wire was 0.20 mm, and the aspect ratio of the wire was 752. As a baseline check, Penney and Jefferson obtained data for free convection (no oscillations) only. The experimental data were correlated by Morgan [51] for $0.25 \leq \text{Gr}_D\text{Pr} \leq 30$ and is displayed below

$$\text{Nu}_D = 1.08(\text{Gr}_D\text{Pr})^{0.213} \quad (2.46)$$

In 1967, Saville and Churchill [63] did a theoretical and analytical study of laminar free convection in boundary layers of horizontal cylinders. They found that for $\text{Pr} = 0.7$

$$\text{Nu}_D = 0.548 \text{Ra}_D^{1/4} \quad (2.47)$$

Saville and Churchill also investigated the limits for which Pr approaches 0 and infinity [12]. For $\text{Pr} \rightarrow 0$

$$\text{Nu}_D = 0.599 \text{Ra}_D^{1/4} \quad (2.48)$$

and for $\text{Pr} \rightarrow \infty$

$$\text{Nu}_D = 0.518 \text{Ra}_D^{1/4} \quad (2.49)$$

Also in 1967, Mabuchi and Tanaka [46] studied the effect of vibration on natural convection on horizontal wires. The fluids tested were air, water, and ethylene glycol. An experiment on the natural convection only was conducted for cylinders with diameters between 0.03 and 0.20 mm and with aspect ratios ranging from 737 to 4790. For $5 \times 10^{-3} \leq \text{Gr}_D\text{Pr} \leq 3$, the following correlation holds

$$\text{Nu}_D = 1.02(\text{Gr}_D\text{Pr})^{0.10} \quad (2.50)$$

Li and Parker [45], in 1967, investigated the effects of acoustics on the natural convection from horizontal wires in water. A wire with a diameter of 0.20 mm was the test specimen. For the case with no acoustic effects and for $5 \leq \text{Gr}_D\text{Pr} \leq 61$, Morgan [51] correlated the results into the following equation

$$\text{Nu}_D = 0.35(\text{Gr}_D\text{Pr})^{0.32} \quad (2.51)$$

In 1968, Bansal and Chandna [5] published an article on free convection from horizontal cylinders. The investigators claim that the new correlation which is presented is valid for any fluid. The data are good for Gr_DPr between 10^{-5} and 10^{10}

$$\begin{aligned} & [\log(\text{Nu}_D)]^2 + \left[\frac{a \log(\text{Gr}_D\text{Pr}) + d}{b} \right] \log(\text{Nu}_D) \\ & + \frac{1}{b} \log(\text{Gr}_D\text{Pr}) [c + \log(\text{Gr}_D\text{Pr})] + \frac{e}{b} = 0 \end{aligned} \quad (2.52)$$

where $a = -26.9268$, $b = 80.3767$, $c = -11.3983$, $d = 94.5623$, and $e = 1.9590$.

In 1968, Weder [71] performed natural convection experiments on horizontal cylinders in sodium hydroxide. The diameter of the cylinder was 26.2 mm, and the

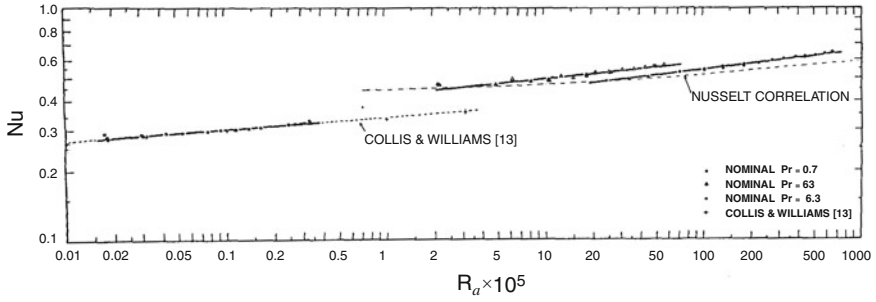


Fig. 2.1 Average Nusselt numbers versus Rayleigh number for $Pr = 0.7, 6.3,$ and 63

aspect ratio was 4. For $6 \times 10^3 \leq Gr_D Pr \leq 6 \times 10^6$, the following correlation is given

$$Nu_D = 0.858(Gr_D Pr)^{0.22} \quad (2.53)$$

Hatton et al. [30] investigated both forced and natural convection low-speed airflow over horizontal cylinders in 1970. The cylinders tested had $0.10 \text{ mm} \leq D \leq 1.26 \text{ mm}$ and $96 \leq L/D \leq 1190$. The investigators, experimentally determined the average Nusselt numbers for the case of no forced airflow (natural convection only) for $4 \times 10^{-3} \leq Gr_D Pr \leq 10$

$$Nu_D = 0.525 + 0.422(Gr_D Pr)^{0.315} \quad (2.54)$$

In 1970, Gebhart, Audunson, and Pera [28, 29] published a series of articles on natural convection from long horizontal cylinders in air with a diameter of 0.01 mm and for $996 \leq L/D \leq 16200$ and for $Pr = 0.7, 6.3$ and 63 . The experimental results are summarized in Fig. 2.1 from [29].

2.2.2 Modern Developments

Around the same time as the review article on Morgan [51] appeared, Churchill and Chu [9] published an article on laminar and turbulent natural convection from a horizontal cylinder. For the laminar regime, they took the limiting Nusselt number expressions of Saville and Churchill [63], seen in Eqs. (2.48 and 2.49), and used a form suggested by Churchill and Usagi [10] as well as considered the limiting value of the Nusselt number found by Tsubouchi and Masuda [67] in order to obtain the following expression for the average Nusselt number for horizontal cylinders

$$\text{Nu}_D = 0.36 + 0.518 \left(\frac{\text{Gr}_D \text{Pr}}{[1 + (0.559/\text{Pr})^{9/16}]^{16/9}} \right)^{1/4} \quad (2.55)$$

The preceding correlating equation matches well with experimental data for all Prandtl numbers in the range of $10^{-6} \leq \text{Gr}_D \text{Pr} \leq 10^9$. The equation, however, did not match well with the experimental data for very low values of $\text{Gr}_D \text{Pr} \leq 10^{-6}$ which were the experimental data of Collis and Williams [13]. Another correlation, good for $10^{-11} \leq \text{Gr}_D \text{Pr} \leq 10^9$, was also developed by Churchill and Chu and is shown below

$$\text{Nu}_D^{1/2} = 0.60 + 0.387 \left(\frac{\text{Gr}_D \text{Pr}}{[1 + (0.559/\text{Pr})^{9/16}]^{16/9}} \right)^{1/6} \quad (2.56)$$

Nakai and Okazaki [52], in 1975, performed an analytical study on natural convection from small horizontal wires for low values of the Grashof number using an asymptotic matching technique. The results were compared with the experimental results of Collis and Williams [13] and valid for $10^{-9} \leq \text{Gr}_D \leq 10^{-1}$

$$\frac{2}{\text{Nu}_D} = \frac{1}{3} \ln E - \frac{1}{3} \ln \left(\frac{\text{Nu}_D \text{Gr}_D}{16} \right) \quad (2.57)$$

where

$$E = 3.1(\text{Pr} + 9.4)^{1/2} \text{Pr}^{-2} \quad (2.58)$$

In 1976, Kuehn and Goldstein [38] presented a correlation for natural convection heat transfer from a horizontal cylinder which is valid at any Rayleigh and Prandtl number

$$\frac{2}{\text{Nu}_D} = \ln \left[1 + \frac{2}{\left[\left\{ 0.518 \text{Ra}_D^{1/4} \left[1 + \left(\frac{0.559}{\text{Pr}} \right)^{3/5} \right]^{-5/12} \right\}^{15} + (0.1 \text{Ra}_D^{1/3})^{15} \right]^{1/15}} \right] \quad (2.59)$$

In 1977, Fand et al. [22] conducted an experimental study for natural convection heat transfer from horizontal cylinders to air, water, and silicone oils ($0.7 \leq \text{Pr} \leq 3090$) for $2.5 \times 10^2 \leq \text{Gr}_D \text{Pr} \leq 2 \times 10^7$. The investigators correlated their data according to which temperature the fluid properties were evaluated. For the fluid properties evaluated at the mean film reference temperature, the correlation is

$$\text{Nu}_D = 0.474 \text{Ra}_D^{0.25} \text{Pr}^{0.047} \quad (2.60)$$

In 1979, Fujii et al. [26] performed a numerical analysis of natural convection about an isothermal horizontal cylinder for $10^{-4} \leq Gr_D \leq 10^4$ and $Pr = 0.7, 10, \text{ and } 100$.

$$\frac{2}{Nu_D} = \ln \left[1 + \frac{4.065}{C(Pr)Ra_D^m} \right] \quad (2.61)$$

where

$$m = \frac{1}{4} + \frac{1}{10 + 4 Ra_D^{1/8}} \quad (2.62)$$

and

$$C(Pr) = \frac{0.671}{[1 + (0.492/Pr)^{9/16}]^{4/9}} \quad (2.63)$$

The authors in [26] claim the above expression agrees well with experimental data in the range $10^{-10} \leq Gr_D Pr \leq 10^7$.

In 1980, Kuehn and Goldstein [39] solved the complete Navier-Stokes and energy equations for natural convection heat transfer from a horizontal isothermal cylinder, which allowed for full plume development. The authors state that previous solutions including boundary-layer assumptions and asymptotic matching solutions are not accurate over the range of $10^0 \leq Gr_D Pr \leq 10^7$. Kuehn and Goldstein employed a finite-difference overrelaxation technique to solve the equations numerically. The data are presented in Table 2.1. In the table, theta is the angular coordinate. See Fig. 2.2 for a schematic diagram of the relevant angles for horizontal cylinders. The data were experimentally verified by Kuehn and Goldstein.

Farouk and Güçeri [24], in 1981, numerically solved for laminar heat transfer from horizontal cylinders. They compared their results with the results of Kuehn and Goldstein [39] and found good agreement.

In 1982, Fujii et al. [27] performed analytical and experimental studies on a thin, horizontal wire. The experimental study was done on a platinum wire that was 0.470 mm in diameter and 238 and 334 mm long situated in air ($Pr = 0.7$). The experimental results, for $Ra_D = 0.37$, which were read from the figure in [27] are listed in Table 2.2.

Furthermore, Fujii et al. adjusted the correlation (Eqs. 2.61–2.63) found in [26] to match their present experimental data. The new correlation is as follows

$$\frac{2}{Nu_D} = \ln \left[1 + \frac{3.3}{C(Pr)Ra_D^m} \right] \quad (2.64)$$

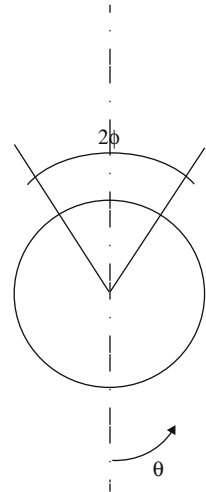
where

$$m = \frac{1}{4} + \frac{1}{10 + 5 Ra_D^{0.175}} \quad (2.65)$$

Table 2.1 Local and average Nusselt numbers from Kuehn and Goldstein [39]

Ra_D^*	Pr	Nu_θ							Nu_D
		$\theta = 0^\circ$	30°	60°	90°	120°	150°	180°	
10^0	0.7	1.41	1.37	1.25	1.08	0.87	0.68	0.56	1.04
10^1	0.7	1.83	1.79	1.67	1.47	1.21	0.94	0.81	1.40
10^2	0.7	2.71	2.66	2.51	2.23	1.80	1.27	0.97	2.05
10^3	0.7	3.89	3.85	3.72	3.45	2.93	2.01	1.22	3.09
10^4	0.7	6.24	6.19	6.01	5.64	4.85	3.14	1.46	4.94
10^5	0.7	10.15	10.03	9.65	9.02	7.91	5.29	1.72	8.00
10^6	0.7	16.99	16.78	16.18	15.19	13.60	9.38	2.12	13.52
10^7	0.7	29.41	29.02	27.95	26.20	23.46	16.48	2.51	23.32
10^4	0.01	3.63	3.56	3.17	2.51	1.74	1.13	0.93	2.40
10^4	0.1	5.25	5.16	4.89	4.34	3.26	1.84	1.12	3.78
10^4	1.0	6.40	6.33	6.10	5.69	4.91	3.36	1.48	5.06
10^4	5.0	6.89	6.82	6.59	6.19	5.55	4.35	1.74	5.66
10^4	10.0	7.01	6.93	6.69	6.29	5.71	4.67	1.79	5.81

Fig. 2.2 Angles of horizontal cylinders



and

$$C(\text{Pr}) = \frac{0.671}{[1 + (0.492/\text{Pr})^{9/16}]^{4/9}} \tag{2.66}$$

In 1983, de Socio [17] experimentally investigated laminar free convection about horizontal cylinders that were partly isothermal and partly adiabatic. The motivation for this study was to determine the heat transfer around metal tubes which were

Table 2.2 Local Nusselt numbers from Fujii et al. [27]

Ra _D	Pr	Nu _θ						
		θ = 0°	30°	60°	90°	120°	150°	180°
0.37	0.7	1.0	0.98	0.95	0.90	0.80	0.70	0.65

partially covered by snow or ice or around tubes that had an internal layer of deposits that affected heat transfer performance. Three cylinders were tested, each having a diameter of 37 mm and a length of 0.5 mm. Referring to Fig. 2.2, in two of the cylinders, a wedge of either $\phi = 45^\circ$ or 90° was removed and replaced with an insulated Teflon (adiabatic) section. The experiments were performed for $1.5 \times 10^4 \leq Gr_D Pr \leq 6 \times 10^5$ and $Pr = 0.7$. The results are correlated with the following equation.

$$Nu_D = B(Gr_D Pr)^m \quad (2.67)$$

where B and m are defined in Table 2.3.

Al-Arabi and Khamis [2] experimentally determined average Nusselt numbers for air ($Pr = 0.7$). The isothermal boundary condition was employed using steam condensation.

For laminar flow and $1.08 \times 10^4 \leq Gr_D \leq 6.9 \times 10^5$ and $Gr_L Pr \geq 9.88 \times 10^7$, the average Nusselt number is

$$Nu_L = 0.58(Gr_L Pr)^{1/3} Gr_D^{-1/12} \quad (2.68)$$

Al-Arabi and Khamis also experimentally determined local Nusselt numbers for air.

For laminar flow and $1.08 \times 10^4 \leq Gr_D \leq 6.9 \times 10^5$ and $Gr_x Pr \geq 1.63 \times 10^8$, the local Nusselt number is

$$Nu_x = 0.58(Gr_x Pr)^{1/3} Gr_D^{-1/12} \quad (2.69)$$

Also in 1982, Farouk and Güçeri used the k - ϵ turbulence model to solve the turbulent natural convection about a horizontal cylinder. Numerical results are presented for $Pr = 0.721$ and $5 \times 10^7 \leq Gr_D \leq 10^{10}$. The authors compared their data to the

Table 2.3 Coefficients and exponents for Eq. (2.67)

ϕ	B	m
0 (isothermal)	0.488	0.146
45	0.543	0.239
90 (adiabatic top)	0.581	0.241
90 (adiabatic bottom)	0.569	0.236

correlations of Churchill and Chu (Eq. 2.56) and Kuehn and Goldstein (Eq. 2.59) and found good agreement with the correlation of Kuehn and Goldstein.

Wang et al. [70], in 1990, numerically computed the natural convection heat transfer from a horizontal cylinder with differing boundary condition using a spline fractional step method. Unlike previous researchers, Wang et al. solved for both the boundary layer and the resultant plume. The investigators present three cases: uniform heat flux (which will be presented in the next section), isothermal surface (which will be presented here), and mixed boundary conditions. The results for the isothermal surface are presented in Table 2.4.

The results in Table 2.4 are in good agreement with those of Kuehn and Goldstein [39] which are displayed in Table 2.1.

In 1993, Saitoh et al. [62] set forth to find benchmark solutions for the natural convection heat transfer around a horizontal isothermal cylinder. The authors of [62] employed five different kinds of numerical methodologies: (a) the ordinary explicit finite-difference method (FDM), (b) the multi-point FDM with uniform mesh, (c) the multi-point FDM with two computation domains, (d) the multi-point FDM with logarithmic coordinate transformation, and (e) the multi-point FDM with logarithmic coordinate transformation and a solid boundary condition. The results for an isothermal boundary condition are listed in Table 2.5 for the multi-point FDM with logarithmic coordinate transformation and a solid boundary condition finite-difference scheme.

The results in Table 2.5 agree well with the results of Kuehn and Goldstein and Wang et al.

Table 2.4 Local and average Nusselt numbers from Wang et al. [70]

Ra_D	Nu_θ							Nu_D
	$\theta = 0^\circ$	30°	60°	90°	120°	150°	180°	
10^3	3.86	3.82	3.70	3.45	2.93	1.98	1.20	3.06
10^4	6.03	5.98	5.80	5.56	4.87	3.32	1.50	4.86
10^5	9.80	9.69	9.48	8.90	8.00	5.80	1.94	7.97
10^6	16.48	16.29	15.95	14.85	13.35	10.58	2.52	13.46
10^7	28.27	27.98	26.95	25.40	23.00	19.68	4.20	23.29
2×10^7	33.46	33.07	31.92	30.07	27.18	23.38	5.42	27.58

Table 2.5 Local and average Nusselt numbers from Saitoh et al. [62]

Ra_D	Nu_θ							Nu_D
	$\theta = 0^\circ$	30°	60°	90°	120°	150°	180°	
10^3	3.813	3.772	3.640	3.374	2.866	1.975	1.218	3.024
10^4	5.995	5.935	5.750	5.410	4.764	3.308	1.534	4.826
10^5	9.675	9.557	9.278	8.765	7.946	5.891	1.987	7.898

Table 2.6 Local and average Nusselt numbers from Chouikh et al. [8]

Ra_D	Nu_θ			Nu_D
	$\theta = 0^\circ$	90°	180°	
10^1	1.789	1.417	0.803	3.024
10^2	2.681	2.197	0.913	4.831
10^3	3.821	3.392	1.219	3.029
10^4	6.023	5.433	1.539	4.831
10^5	9.694	8.798	1.991	7.911
10^6	16.12	14.948	2.144	13.216

Chouikh et al. [8], in 1998, expanded the work of Saitoh et al. [62]. They numerically solved the conservation of mass, momentum, and energy equations via the vorticity-stream function approach. Their results for $10^1 \leq Gr_D Pr \leq 10^6$ are shown in Table 2.6.

Recently, in 2009, Atayilmaz and Teke [3] performed experimental and numerical studies on natural convection from horizontal cylinders. The authors in [3] claim that although the subject has been studied extensively for over 50 years, discrepancies in all of the data still exist due to various factors. Further, the motivating application of Atayilmaz and Teke is heat exchangers used in small refrigeration applications ($D = 4.8$ and 9.45 mm) and the authors state that there has been no investigation of this particular diameter. Based on the experimental data, the following correlation was proposed for $7.4 \times 10^1 \leq Gr_D Pr \leq 3.4 \times 10^3$ and $Pr = 0.7$

$$Nu_D = 0.954(Gr_D Pr)^{0.168} \quad (2.70)$$

Atayilmaz and Teke compared their experimental results with that of Morgan [51], Churchill and Chu [9], and Fand and Brucker [23] and found agreement to within 20%. For the numerical portion, the authors compared their results to that of Merkin [50] who also did a numerical simulation. The two sets of results follow the same trend, but are not in good agreement.

2.3 Heat Flux Boundary Conditions

According to Dyer [16], up until his current investigation in 1965, studying natural convection from horizontal cylinders with a uniform heat flux boundary condition appeared to be neglected in the literature. Therefore, Dyer did both a theoretical and an experimental study. The theoretical study, which is valid for $10^3 \leq Gr_D^* Pr \leq 10^{10}$ yielded

$$Nu_D = 0.61(Gr_D^* Pr)^{0.192} \quad (2.71)$$

The experimental results, which were conducted in air using an electrically heated horizontal cylinder that was 7.7 cm in diameter, validated the analytical results.

In 1972, Wilks [72] did a theoretical study of natural convection from two-dimensional bodies with constant heat flux using the boundary-layer approximation. The results in [72] were correlated by Churchill [11] into the following equation

$$Nu_D = 0.579 \left(\frac{Gr_D Pr}{[1 + (0.442/Pr)^{9/16}]^{16/9}} \right)^{1/4} \tag{2.72}$$

The values of the average Nusselt number were obtained by averaging the local values calculated by Wilks [72]. Churchill and Chu [12] decided to leave this expression in terms of $Gr_D Pr$ instead of $Gr_D^* Pr$ in order to show that the dependence of Nu_D on $Gr_D Pr$ was the same for both isothermal and uniform heat flux situations.

In 1987, Qureshi and Ahmad [58] performed a numerical solution of the full Navier-Stokes and energy equations for a uniform heat flux cylinder in air ($Pr = 0.7$). The results are presented in Table 2.7 for $10^{-2} \leq Gr_D^* Pr \leq 10^7$.

Qureshi and Ahmad correlated the results for $10^0 \leq Gr_D^* Pr \leq 10^7$ and $Pr = 0.7$ with the following equation.

$$Nu_D = 0.800(Gr_D^* Pr)^{0.175} \tag{2.73}$$

For $Gr_D^* Pr \leq 10^0$, the average Nusselt numbers may be predicted by correlations for isothermal cylinders.

In 1990, Wang et al. [70] numerically computed the natural convection heat transfer from a horizontal cylinder with differing boundary condition using a spline fractional step method. The results for the uniform heat flux case are shown in Table 2.8.

Wang et al. [70] compared their results with Wilks [72], Churchill [11], and Qureshi and Ahmad [58] and found good agreement.

Table 2.7 Local and average Nusselt numbers from Qureshi and Ahmad [58]

Ra_D^*	Nu_θ							Nu_D
	$\theta = 0^\circ$	30°	60°	90°	120°	150°	180°	
10^{-2}	0.48	0.47	0.47	0.46	0.53	0.45	0.45	0.46
10^{-1}	0.70	0.70	0.69	0.68	0.68	0.67	0.67	0.68
10^0	0.93	0.92	0.90	0.86	0.82	0.79	0.78	0.86
10^1	1.34	1.32	1.28	1.20	1.11	1.03	1.00	1.19
10^2	2.06	2.05	1.99	1.88	1.70	1.48	1.38	1.80
10^3	3.03	3.02	2.96	2.83	2.59	2.17	1.88	2.67
10^4	4.50	4.47	4.39	4.23	3.94	3.30	2.57	3.98
10^5	6.76	6.72	6.58	6.35	5.95	5.10	3.50	5.97
10^6	10.23	10.16	9.95	9.59	9.04	7.84	4.85	9.02
10^7	15.50	15.39	15.08	14.57	13.77	12.07	6.88	13.70

Table 2.8 Local and average Nusselt numbers from Wang et al. [70]

Ra_D^*	Nu_θ							Nu_D
	$\theta = 0^\circ$	30°	60°	90°	120°	150°	180°	
10^6	9.87	9.83	9.60	9.24	8.94	7.91	5.02	8.88
10^7	15.04	15.00	14.72	14.08	13.58	12.28	7.14	13.57
10^8	23.12	22.84	22.57	21.92	20.85	19.63	10.87	21.00
2.5×10^8	28.05	27.34	26.98	26.18	25.01	23.17	12.26	25.08

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