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Sliding-Mode Observers Based on Equivalent Control Method

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Abstract

In the paper the sliding-mode observer for nonlinear system is proposed. The observer is based on the equivalent control method. Comparing with the known output global linearization approach it is given in terms of of the system variables and doesnot require nonlinear state transformation. Several examples are presented which illustrates the proposed method.

I. Introduction

The problem of state estimation for nonlinear system

$$\dot{x} = f(x), \tag{1}$$

where $x \in R^n$ by output vector measurements

$$y = h(x) \in R^m \tag{2}$$

was considered by many authors (see State-of-the-Art Survey by Misava and Hedrick [3]). For exactly known smooth nonlinearities f and h the method of global linearization permits by using nonlinear state transformation

$$x = T(z) \tag{3}$$

to obtain a system in the new variables $z \in R^n$ which is linear with respect to z. Then the standard linear pole-placement technique can be used for designing observer. For successfull application of this method the nonlinear transformation T as well as its inverse T^{-1} must be determined.

The main idea of the observer design in the presented paper is in using equivalent control method to obtain additional information from the system by equivalent values of discontinuous function in sliding-mode. For linear case in [1], [4] a sliding-mode observer was considered which is based on sequential application the state transformations reducing the system to the block form. The presented algorithm although similar in this case doesnot require state transformations.

II. Observer design.

It will be considered the case of scalar observations ($m = 1$).
Let's introduce the following notations:

$$H(x) = col(h_1(x), \ldots, h_n(x)), \tag{4}$$

where

$$h_1(x) = h(x), \tag{5}$$

$$h_i(x) = \frac{\partial h_{i-1}(x)}{\partial x} f(x), \tag{6}$$

$i = 1, \ldots, n$.
Actually $h_{i+1}(x)$ is ith Lie (directional) derivative of the function $h(x)$ along the trajectories of the system (1)

$$h_i(x) = L_f^{i-1} h(x). \tag{7}$$

From (1) and (5),(6) we have that if $x(t)$ is a solution of (1) then

$$\frac{d}{dt} h_i(x(t)) = h_{i+1}(x(t)), \tag{8}$$

$i = 1, \ldots, n-1$.

The assumptions imposed on the functions $f(x)$ and $h(x)$ are as follows:

Assumption 1. Functions $f(x)$ and $h(x)$ are smooth enough in order all the partial derivatives introduced above exist and are continuous.

Assumption 2. For a given domain $X_0 \subset R^n$ of initial conditions of the system (1), which is assumed to be bounded, all the solutions of the system (1) belong to the open one-component domain $X \subset R^n$, for all $0 \leq t < \infty$. Jacobian of the continuous map

$$H : R^n \mapsto R^n \tag{9}$$

$H(x) = col(h_1(x), \ldots, h_n(x))$, is nondegenerating in X i.e.:

$$\left|\det \frac{\partial H(x)}{\partial x}\right| \geq \delta > 0 \qquad (10)$$

for some $\delta > 0$ and every $x \in X$.

From **Assumption 2** it follows that the map H is an injection. So it is one to one correspondence from the domain X to $H(X)$.
This assumption is actually an observability condition for (1),(2).
To estimate system (1) state variables by measurements (2) we will use an observer of the form:

$$\dot{\hat{x}} = \left(\frac{\partial H(\hat{x})}{\partial x}\right)^{-1} M(\hat{x})sgn(V(t) - H(\hat{x})), \qquad (11)$$

where $V(t) = col(v_1(t), \ldots, v_n(t))$, $sgn(z) = col(sgn(z_1), \ldots, sgn(z_n))$

$$v_1(t) = y(t), \qquad (12)$$

$$v_{i+1}(t) = (m_i(\hat{x})sgn(v_i(t) - h_i(\hat{x}(t)))_{eq}, i = 1, \ldots, n-1, \qquad (13)$$

and matrix M is $n \times n$ diagonal matrix with positive elements

$$M(\hat{x}) = diag(m_1(\hat{x}), \ldots, m_n(\hat{x})). \qquad (14)$$

It will be shown that by suitable choice of M observer (11) converges in **any** prescribed finite time interval.

Theorem 1. Under **Assumptions 1** and **2** for any $t_1 > 0$ there exists a diagonal matrix $M(\hat{x})$ such that $\hat{x}(t) \equiv x(t)$ for $t \geq t_1$.

Proof. Since the map $H : X \mapsto R^n$ is an injection it is sufficient to show that $e(t) = H(x(t)) - H(\hat{x}(t))$ converges to zero in finite time.
From (1),(2) and (11) it follows that

$$\dot{e}(t) = \frac{dH(x(t))}{dt} - \qquad (15)$$

$$\frac{\partial H(\hat{x}(t))}{\partial x}\left(\frac{\partial H(\hat{x}(t))}{\partial x}\right)^{-1} M(\hat{x})sgn(V(t) -$$

$$H(\hat{x}(t))) = \frac{dH(x(t))}{dt} - M(\hat{x})sgn(V(t) - H(\hat{x}(t))).$$

Therefore from (8)

$$\dot{e}_i(t) = h_{i+1}(x(t)) - m_i(\hat{x})sgn(v_i(t) - h_i(\hat{x}(t))) \quad (16)$$

for $i = 1, \ldots, n$.
If $|h_2(x(t))| \leq m_1(\hat{x}(t))$ then in the first equation (16) sliding mode occurs since $v_1(t) = h_1(x(t))$ and hence $v_1(t) - h_1(\hat{x}(t)) = e_1(t)$.
During the sliding mode $e_1(t) = 0$ and according to the equivalent control method $(m_1(\hat{x})sgn(v_1(t) - h_1(\hat{x}(t)))_{eq} = h_2(x(t))$ or as it follows from (13) $v_2(t) = h_2(x(t))$. Again if the condition $|h_3(x(t))| \leq m_2(\hat{x}(t))$ is satisfied sliding mode occurs in the second equation and we will have $e_2(t) = 0$ and so on.
When the sliding mode occurs in the last equation (16) we have $e(t) = 0$. The conditions for sliding mode to exist in every equation (16) are

$$|h_{i+1}(x(t))| \leq m_i(\hat{x}(t)). \qquad (17)$$

From boundedness of the initial condition region X_0 follows that all solutions $x(t)$ started in X_0 are uniformely bounded on any finite time interval $[0, T]$. Therefore by increasing m_i any desired time $t_1 < T$ of convergence $\hat{x}$ to x can be achieved. After the moment t_1 since $\hat{x}(t) = x(t)$ this inequality can also be maintained by appropriate choice of m_i.

Remark. The choice of matrix M is defined by the region of initial conditions X_0 for the system (1) and the upper estimates of h_i.

References

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