Characterizing Mathematical Digital Literacy: A Preliminary Investigation

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This preliminary report offers initial results from a study designed to begin identifying characteristics of digital literacy in mathematics. Undergraduate students in a three-course honors calculus sequence were provided with tablet computers as part of a digital literacy initiative and digital tasks were integrated into the courses. Student work was analyzed and coded for type of ICT tool use and possible components of mathematical digital literacy. The specific types of tasks developed for and integrated into the class will be discussed below with specific illustrative examples highlighted. The aspects of mathematical digital literacy illuminated by student work will be outlined, with some initial conclusions and conjectures about the nature of digital literacy in mathematics.

Introduction and Background

The ever-increasing role of technology in everyday life and work prompts questions about the skills and understandings needed for effective use of that technology. The range of information and communication technology (ICT) tools grows ever greater, and the ability to obtain, manage, synthesize, analyze, and communicate information is constantly changing and adapting. As technological capabilities rapidly change, the accompanying skills and understandings necessarily shift in response. Competence and knowledge with technological tools is described by and named with a variety of terms, the most prevalent of which is *digital literacy*, a term first defined by Gilster (1997) as "the ability to understand and use information in multiple formats from a wide range of sources when it is presented via computers" (p. 1). It is now frequently used as an umbrella term with a variety of implications, though there is general agreement that digital literacy involves interaction and integration of a number of proficiencies, such as procedural competence with ICT tools, cognitive skills for using them effectively, and social and communication skills (Avriam & Eshet-Alkalai, 2006; Goodfellow, 2011). The use of the word "digital" is itself far from universal, with some sources variously referring to media literacy, digital and media literacy, ICT literacy, or related specialized terms. This paper will use the term "digital literacy" to encompass the wide variety of terms used in order to draw on the valuable contributions of multiple approaches.

Educational Testing Service (2003) characterized seven proficiencies that characterize general digital literacy: Define, Access, Manage, Integrate, Evaluate, Create, and Communicate. Specific applications of the term might include or alter those proficiencies within the context of a particular subject. In education, digital literacy has been increasingly emphasized in general (Gutierrez & Tyner, 2012) and in mathematics specifically (NCTM, 2000, National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), and has been shown to have a positive impact on student learning (Li & Ma, 2010). ICT tools are also increasingly integrated into the work of research mathematics (Monroe, 2014).

Despite the increased emphasis on and integration of ICT tools within mathematics, *mathematical digital literacy* is not well-defined. The competencies with ICT tools specific to mathematics would be of particular concern to educators, curriculum developers, and many other stakeholders within the field. This presentation describes an investigation into digital literacy among undergraduate students in an honors calculus sequence. By assessing how students engaged with digital tools that were often new and unfamiliar in order to solve mathematical problems and understand mathematical concepts, preliminary characteristics of mathematical digital literacy emerge.

Context and Methodology

Setting and Data Collection

Undergraduates in a three-course honors calculus sequence were provided with tablet computers as part of a digital literacy initiative at their university. These courses (Honors Calculus I - 22 students; Honors Calculus II - 22 students; Honors Calculus III - 18 students) covered the traditional material of the calculus sequences in a "late transcendentals" ordering. In the past, the mathematics program had not emphasized the use of digital tools, so integrating them into the work of the course provided an opportunity to observe emergent digital literacy in mathematics and investigate an initial characterization. The primary digital tools introduced to the students by the instructor were Wolfram Mathematica and the online Desmos graphing calculator.

An initial assignment allowed students to use any tools they might choose and consisted of problems for which digital ICT might be useful, but which focused on concepts already familiar to students. For example, finding the zeros of a sixth degree polynomial or determining the domain of a ratio of logarithmic functions. This served as an initial assessment of how students chose to use such tools. Throughout the semester, two types of digital tasks were used to assess student interaction with and use of ICT tools - digital assignments and digital exams. These were supplementary to the traditional written course content.

Digital assignments were primarily meant to provide students with base-line experience using digital tools to solve mathematical problems and were typically assigned as handouts or pdf files related to the content that had recently been discussed in lecture. A set of instructions led students through the use of Mathematica or Desmos (depending on the content) to visualize and solve a set of problems. Often the instructions would require students to choose parameters to create their own individualized problems. For problems that required more advanced coding, students would be provided with a template file to edit. During in-class digital assignments, the instructor would typically provide demonstrations and move around the classroom to help students with syntax and interpretation. Students submitted their work digitally as either a Mathematica Notebook file or a link to a Desmos graph. Though an indirect consequence of using these ICT tools may have been an increase in student understanding of content, the primary focus of digital assignments was on gaining literacy with digital tools and accessing new problems and information via their use.

Digital exams were completed in the class period following a written exam. The problems on the digital exam often required students to create digitally generated images/animations and to make computations that could not be completed by hand in a reasonable amount of time. The exams were "open resource" - students were allowed to use any digital resource at their disposal except for online help forums, including those not discussed by the instructor. This permitted analysis of the ways in which they chose to use digital tools. The TPACK framework (Mishra & Koehler, 2006) and Howland, Jonassen, and Marra's (2012) five dimensions of learning involving ICT tools served as general guides for designing the integration of digital tools into assignments and exams in each course.

Student work was collected for each digital assignment and digital exam. On the initial assignment and digital exams, students were asked to identify what digital tools they used. At the end of the course, a survey about their attitudes toward and use of digital tools gave additional information. The first digital assignment and the digital exams permitted students the most freedom in selecting and utilizing digital tools and were therefore the first to be analyzed for characteristics of digital literacy. Open coding (Strauss & Corbin, 1990) was used to develop coding schemes that described student use of and interaction with the digital tools.

Examples of Digital Tasks

An example of a digital assignment. Early in the Calculus I course, students were required to complete a digital "Desmos" assignment asking them to explore limits involving trigonometric functions. The first portion of the assignment asked students to consider the limit $\lim_{x\to 0}\frac{sin(ax)}{bx}$ using the function as graphed in an existing Desmos file. Students used the Desmos "sliders" to evaluate the limit for various values of *a* and *b*. Eventually, students choose their own unique values to verify the pattern. The assignment included a similar exploration for 3 other common trigonometric limits: $\lim_{x\to 0}\frac{ax}{sin(bx)}$, $\lim_{x\to 0} ax\ cot(x)$, and $\lim_{x\to 0}\frac{1-cos(ax)}{bx}$.

Apart from the digital assignment, students were provided with a rigorous proof of the first computation. As with many digital assignments, this one provided exploration, experience, and visualizations that would later support formal computations, theorems, and proofs. During the assignment, the instructor reminded students that the use of Desmos itself as a tool was only the secondary purpose of the assignment. The primary purpose of this assignment was to encourage students to begin using technology when presented with an apparently intractable problem.

An example of a digital exam. Figure 1 shows a Calculus II Digital Exam problem on Taylor series that is impractical to solve by hand. To illustrate the ways in which students used digital tools to solve problems and communicate their solutions, samples of student work on this problem are included in Figure 2 below and discussed later.

3. (5 pts) $g(x) = \int \sin(x^2)\,dx$ is not an elementary function but still has a convergent Taylor series. Assume $g(0) = 0$. Find the 15th Taylor polynomial centered at $x = 0$ of this funciton. Use this polynomial to approximate $g(3/2)$ (even though we don't have a formula for g to "plug in").

Figure 1: A Calculus II Digital Exam Problem

The nature of the problems in both types of tasks was varied in order to expose students to different ways ICT tools might be useful and to highlight the various ways students chose to use them.

Preliminary Results

The full analysis of student work provided rich data on use of ICT tools. Below, a very brief overview of the initial results is given. One specific example is used to illustrate how the digital tasks highlighted the variety of uses of digital tools and how the data led to this preliminary characterization of mathematical digital literacy. The proposed presentation would incorporate more examples in order to draw the connections more completely.

Notable Results from Surveys

Thirty-five students responded to the post-course survey. When asked to rate their comfort level with digital tools at the beginning and end of the semester on a scale of 1 (not comfortable at all) to 10 (very comfortable), every student reported the same or greater levels of confidence. The mean change in self-reported comfort level was 1.6 with a median change of 1. All students reported using some technology outside of the digital assignments and course requirements.

When students were asked to describe how they used digital tools in the class, the most common response was for visualization. In particular, students noted the value of Desmos for graphing equations and of Mathematica for graphing three-dimensional solids. They also valued the ability to quickly perform calculations and to check answers, though many noted that learning the syntax for Mathematica was difficult, at least initially.

An Example of Results From Student Work

Analysis of student work on the digital tasks illustrated the different ways in which students engaged with digital tools. For one instance, two examples of student work on the problem from Figure 1 are shown in Figure 2:

Student 1 submission of #3	Student 2 submission of #3

In[1]:= `(* Problem 3 *)`
`p15[x_] = Integrate[Normal[Series[Sin[x^2], {x, 0, 15}]], x]`
`N[p15[3/2]]`
Out[1]= $\frac{x^3}{3}-\frac{x^7}{42}+\frac{x^{11}}{1320}-\frac{x^{15}}{75\,600}$
Out[2]= 0.777928
	`(*3*)`
	`Clear[f, a];`
	`f[x_] := Sin[x^2]`
	`a = 0; (*center*)`
	`Integrate[Normal[Series[f[x], {x, a, 15}]] , x]`
	`p8[x_] = f[a] + Integrate[Normal[Series[f[x], {x, a, 15}]] , x]`
	`(*we must add the constant term which is f(center)*)`
	`N[p8[3/2]] (*approximation for g(1)*)`
	$\frac{x^3}{3}-\frac{x^7}{42}+\frac{x^{11}}{1320}-\frac{x^{15}}{75\,600}$
	$\frac{x^3}{3}-\frac{x^7}{42}+\frac{x^{11}}{1320}-\frac{x^{15}}{75\,600}$
	0.777928

Figure 2: Samples of student work on Calculus II Digital Exam problem #3

Both students chose Mathematica for this particular problem. This is not surprising given the nature of the problem and the tools with which most students were comfortable. What is interesting is the difference in their processes. Student 1 submitted a concise and correct solution. Student 2 also submitted a correct solution, however, the student copied previously used code provided by the instructor to find the 8th Taylor polynomial of a given function. Note that Student 2 did not bother to change p8 to p15 even though the problem is to find the 15th Taylor polynomial. A side-by-side comparison suggests that Student 1's solution exhibited greater digital literacy since they were comfortable enough with the content and syntax to simplify their code whereas Student 2 attempted to simply mimic a previous application of digital tools.

Toward an Understanding of Mathematical Digital Literacy

Students tended to use digital tools in the following major ways:

1. *Determine which tool should be used to solve a given problem*.
2. *Learn and apply syntax of technological tool (sometimes based on template)*.
3. *Decide how to translate mathematics into input in chosen tool*.
4. *Interpret technological results to find a proposed solution.*
5. *Use technology to justify that a proposed solution is correct*.
6. *Display and submit answer and supporting work digitally.*

Though there was much variation in the particular ways students engaged in these activities with ICT tools, they fell into these six main categories of use. Such a categorization permits some initial conjectures about components of mathematical digital literacy:

Component 1: Ability to assess and choose tools based on potential use along multiple proficiencies

Component 2: Translation between digital and mathematical contexts, including multiple representations (notational, graphical, syntactical) and digital and mathematical troubleshooting

Component 3: Using ICT tools to enhance or complement (rather than replace) mathematical understanding

Component 4: Using ICT tools to communicate mathematics

These components are related to the seven proficiencies with ICT tools described by ETS (2003), but are specific to mathematics. A more nuanced and detailed analysis is underway and will be described in greater detail in the proposed presentation.

Conclusion

Work remains to be done to fully characterize digital literacy for mathematics. However, this preliminary study supports the idea that a focus on learning and doing mathematics within digital environments increases student facility and comfort with ICT tools. The ways students utilized digital tools provides some initial indications of important components of digital literacy.

The Proposed Presentation

The proposed preliminary report would include the information summarized in this proposal in addition to more specific examples of student work and more careful and nuanced descriptions of components of digital literacy. As a preliminary report, the authors hope to use this as an opportunity for feedback from experienced and engaged mathematics educators to shape future research and analysis on this subject. In addition to welcoming critical assessment and feedback of this preliminary research, the authors propose the following questions to be considered by the audience:

1. How do we, as a research community, move toward a fuller understanding and description of what digital literacy means in mathematics? What research designs might be useful or beneficial?
2. How does such an understanding remain responsive to changes in availability and capability of digital tools?

3. How might we begin to understand the relationship between mathematical digital literacy, mathematical conceptual understanding, and proficiency with mathematical practices?

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