Developing Autonomous, Targeted Feedback in Precalculus

Project Team:
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1 Proposal Narrative

1.1 Introduction

Feedback is a critical component to learning. There are various levels of feedback:

- Knowledge of Result - Whether the solution is correct or incorrect;
- Knowledge of Correct Result - Providing the correct solution;
- Instruction-based Elaboration - Explanation of the correct solution and/or instruction on the underlying concept; and
- Targeted Elaboration - Explanations based on student response (Melis and Andres, 2005).

Literature suggests that only providing Knowledge of Result and Knowledge of Correct Result can be counterproductive (Narciss and Huth, 2004). Moreover, providing all of the explanation after a response (as opposed to targeted responses or step-by-step responses) may result in a superficial treatment as the student attempts to mimic the explanation for a future question or cognitive overload as the student has too much information to process how it relates to their work (Mayer and Moreno, 2002). Therefore developing ways to provide targeted feedback is crucial to enhance online learning.

One method to leverage technology and provide targeted feedback is through effective distractor generation. A *distractor* is a plausible but incorrect answer to the question. Distractors can be constructed in a number of ways: by content specialists recalling common errors or misconceptions they are familiar with, recording any common errors identified in educational research experiments, or theoretically-predicted errors or misconceptions according to published mathematics education theoretical perspectives (Chamberlain Jr. and Jeter, 2020). Targeted feedback can then be associated to each distractor. Consider solving the linear equation below.

$$\frac{-5x-3}{4} - \frac{-9x-7}{3} = \frac{6x-7}{5}$$

The solution is approximately -5.424. If a student were to divide just the coefficients in front of x by the denominator rather than dividing *both* parts of the numerator by the denominator they would have arrived at -20. If a student did not distribute the negative in front of the second fraction they would have arrived at approximately 3.061. Targeted feedback can now be provided based on two common procedural errors associated with the content objective.

The difficulty of designing effective distractors (and thus targeted feedback) cannot be understated. Gierl et al. (2017) consider distractors to (i) require a significant amount of time and resources to create, (ii) affect item quality and learning outcomes, and (iii) provide diagnostic inferences about students' knowledge. Note that building a free-response question with associated feedback for alternative answers is essentially creating a multiple-choice problem where the student cannot see the alternative choices.

The overriding goal of this project is to investigate student knowledge in a Precalculus course at ERAU-W in order to construct autonomous, targeted feedback for free-response questions to enhance students' online learning. This will be accomplished by analyzing student responses to exam questions and interviewing students to probe how their mathematical conceptions correspond to their exam responses.

1.2 Related Literature and Products

There does exist a category of multiple-choice assessments that proport to assess conceptual understanding: Concept Inventories. *Concept Inventories* associate varying levels of understanding to the options in a question (O'Shea et al., 2016). Concept Inventories were first developed in the context of force for physics (Hestenes et al., 1992), but have since been introduced in mathematics for precalculus concepts (Carlson et al., 2010), calculus concepts (Epstein, 2013), and functions (O'Shea et al., 2016). Creating a concept inventory is a time-consuming matter involving: creation of pilot problems, validation of these problems through quantitative and qualitative methods, and ensuring these questions provide a complete picture of students' understanding over some small set of concepts. The major limitation of these projects is their inability to be administered multiple times due to the static nature by which the question structures and distractors were constructed. While these assessments focus on summative feedback, targeted feedback could be developed based on the varying levels of conception already built into the multiple-choice options.

Feedback based on *how* a student responds to an open-response question is the hallmark feature of Intelligent Tutoring Systems. These systems rely on a cognitive framework for how students think to provide targeted feedback. For example, the Adaptive Control of Thought–Rational (ACT-R) out of Carnegie Mellon posits that each task a human performs can be decomposed into cognitive and perceptual operations (Ritter et al., 2007). Under this view "learning is a process of encoding, strengthening, and proceduralizing knowledge" (p. 250). ACT-R attempts to identify individual knowledge components that inhibit the procedures needed to solve a mathematical problem and an intelligent tutor can provide targeted feedback to assist with the specific procedure.

Utilizing how a student thinks about a concept, how they perceive a question, and what responses the concept and question may provoke to provide targeted feedback is not a common aspect of commercial online homework systems. Some platforms, such as MyLab, rely on free-response questions for homework and large multiple-choice test banks for quizzes/exams. Learning resources normally include a video lecture on the topic at hand, a worked-out solution of a similar problem, and a step-by-step guide having the student's responses checked after each step. An incorrect response to a free-response question would prompt generalized feedback that may or may not be relevant to why a student's answer deviated from the expected response. Moreover, worked-out solutions may promote replicating an outside procedure over reflecting on one's own thought process leading to minimal retention of knowledge.

Adaptive homework systems rarely adapt their feedback based on *how* a student responds. Rather, they rely on a student's correct or incorrect answers to a series of questions linked to specific content objectives to map student understanding. For example, ALEKS relies on Knowledge Space Theory. This theory assumes student knowledge can be deconstructed into a discrete set of knowledge elements (Falmagne et al., 2006). For example, the content objective "Solve a linear equation with rational coefficients" could be decomposed into the collection of knowledge: (1) Solve a linear equation with integer coefficients; Multiply a fraction; and Use the distributive property. The theory proposes that an inability to answer the question stems from a deficiency in one of the elements. Using a stochastic process, an ALEKS assessment could prompt the student to complete a task on individual elements until the system determines the original deficient element. Notice that the system diagnoses the deficiency based solely on whether the student answered a series of questions correctly or not and does not take into consideration *how* the student answered. While this is effective at evaluating where a student is in their learning process, it may not provide the feedback needed to challenge previously held misconceptions or correct over-generalized procedures.

1.3 Theoretical Lens

This study will use a Constructivist lens to examine student understanding. *Constructivism* theorizes that people actively construct knowledge by integrating what they know with interactions in their environment. In this way learning is the perpetual act of building, integrating, or replacing knowledge based on experience. If there is not enough experience, the person may not sufficiently generalize a concept (e.g., every cat they have seen is black and so they conceptualize cats as always having the "black" quality). If the experience is not internalized, then it would not be integrated into the person's knowledge and thus quickly forgotten (e.g., using a mathematical procedure like the Quadratic Formula without questioning what it is and how it works). Students' unique experiences and ways they integrate these experiences make it important to consider *how* a student responds and provide targeted feedback based on that response. Moreover, the individual nature of how knowledge is integrated means that the environment may affect the way a student interprets, conceptualizes, and responds to mathematical concepts. Thus is it important to situate any study on how students think in the student's specific environment.

1.4 Research Questions

1. What are the common responses to midterm exam questions in MATH 111 at ERAU-W?
2. What are the associated conceptions, misconceptions, procedures, or errors with the common responses to midterm exam questions in MATH 111 at ERAU-W?
3. How do students in MATH 111 at ERAU-W conceptualize variables?
4. How do students in MATH 111 at ERAU-W conceptualize functions?

1.5 Methodology

Two sources of data will be collected to answer the research questions. Exam data and scratchwork will be collected from 100 students taking MATH 111 at ERAU-W in the October 2021 term. This data will be deidentified and uploaded to an online qualitative analysis software: Dedoose. Interviews will be completed with 10 students whose exam data was collected. Interviews will be voluntary and pooled from students who are not part of Dr. Chamberlain's classes. Students who volunteer for an interview will receive pseudonyms to connect their exam data and interview data. Interviews will be approximately one hour and have the following three phases:

- Gather general information about the student (year, major, prior math experience, etc);
- Have student review their exam and explain noteworthy responses; and
- Answer questions about variables and functions.

1.6 Analysis Plan

Analysis of thought processes/procedures will follow Grounded Theory process of open, axial, and selective coding. *Open coding* consists of categorizing data without a specific theoretical framework in mind. *Axial coding* draws connections between the codes formed in the open coding process. Finally *selective coding* organizes the collections of codes formed in the axial coding step into a single overriding category. This 3-step process allows researchers to theorize based on data and then compare these theories to established theoretical frameworks.

For the open coding portion, each exam response will be tagged as either "correct" or as a specific distractor. Scratchwork will then be used to determine what underlying thought process or procedure is associated to the specific distractor. Frequencies of correct response and individual distractors will also be cataloged for each exam question.

Interview transcript analysis will consist of two parts. For parts where the student is discussing their underlying thought process during the exam, the underlying conception and associated response will be recorded. Each question's pairings will be analyzed to find any patterns in student thinking and how this affected their response. For parts where the student is answering questions about variables and functions, each student conception and response will be analyzed to determine how their understanding of the overriding concepts affected their response. Patterns in student thinking and overriding conceptions will be compared to research results in a confirmatory fashion or provide avenues to further explore students' understanding of Precalculus at ERAU-W.

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2 Timeline, Outcomes, and Deliverables

2.1 Project Timeline

Month(s)	Activities	Products
Oct '21	- Recruit student researcher; - Develop interview protocol; - Submit to IRB.	- Semi-structured interview questions; - Detailed research protocol. - IRB Approval.
Nov '21	- Collect exam responses and scratchwork; - Begin conducting interviews; - Send interview videos for transcription.	- Quantitative data (exam response and scratchwork); - Qualitative data (6 interviews); - Transcripts of interviews.
Dec '21	- Complete conducting interviews; - Send interview videos for transcription; - Prepare quantiative data for analysis.	- Qualitative data (10 interviews total); - Transcripts of interviews; - Data uploaded/organized in Dedoose; - Links between student transcript and exam responses/scratchwork.
Jan '22 Feb '22 Mar '22 Apr '22	Quantitative and Qualitative analysis of data.	- Frequencies of unique responses per exam question; - Categorized students' conceptions based on exam response explanation; - Categorized students' conception of variables; - Categorized students' conception of functions.
May '22 Jun '22	Disseminate results.	- Manuscript for high-rank journal; - Presentation of results at 1 conference.
Jul '22 Aug '22 Sept '22 Oct '22	Prepare NSF IUSE: EHR grant application.	- In-depth literature review on online homework systems utilizing targeted feedback; - Targeted feedback based on results results of seed project; - Secure additional internal or external collaborator; - Draft of grant.

2.2 Future Work

Results from this study can enhance the research community's understanding of how student conceptions of variables and functions affect their responses in a Precalculus course. This knowledge can be used in conjunction with results on students' conception of equality, covariational reasoning, functions, and other topics to develop targeted feedback to enhance student learning. The frequencies of individual distractors can also provide targets to create additional student resources for success. Moreover, results on learning trajectories for students in MATH 111 at ERAU can inform future MATH 111 course redevelopment initiatives.

This project will lay the foundation to apply for an NSF grant to develop an open-source online homework system for Precalculus that provides targeted feedback as well as designing a digital notebook where students can record ideas and organize their thoughts. The project would involve writing lessons, homework, and assessments on functions. Learning trajectories will be crucial for developing lessons and homework that target ERAU-W student knowledge. Categorized responses to exam questions would inform effective assessment development. Success of the project would continue to serve ERAU-W's top-ranked online teaching.

2.3 Expected Results, Outcomes, and Deliverables

Research Question	Expected Results	Outcomes	Deliverables
What are the common responses to midterm exam questions in MATH 111 at ERAU-W?	- Distractors will align with previously identified distractors; - New distractors specific to ERAU-W students will be identified.	Knowledge on the concepts/procedures ERAU-W MATH 111 students perform well on and concepts/procedures that need more educational resources.	- List of distractors per exam question; - List of conceptual difficulties students encounter; - List of recommended educational supplements for course.
What are the associated conceptions, misconceptions, procedures, or errors with the common responses to midterm exam questions in MATH 111 at ERAU-W?	- Unique conceptions due to aeronautic speciality will appear in exam responses and/or interviews; - Procedural errors will remain similar to literature and previous work.	Knowledge on specific misconceptions, errors, and procedures associated to current MATH 111 exam.	Cataloged list of errors, procedures, conceptions, and misconceptions per exam question.
How do students in MATH 111 at ERAU-W conceptualize variables?	- Students do not have clear distinction between constant, variable, and value. - Underdeveloped conception of variable leads to difficulties in precalculus.	Knowledge on levels of variable conception for students at ERAU-W.	Learning trajectory for variable concept, including where students at ERAU-W were at and where the course wants students' conception to be at.
How do students in MATH 111 at ERAU-W conceptualize functions?	- Students do not have clear distinction between relation, function, and equation. - Students have weak relation between different classes of function. - Student conception of functions relies on general shape of function and not as a relation of how two quantities change.	Knowledge on levels of function conception for students at ERAU-W.	Learning trajectory for function concept, including where students at ERAU-W were at and where the course wants students' conception to be at.

3 Prior Work

3.1 Automatic and Diagnostic Item Generation (2018-present)

Project team: Dr. Darryl Chamberlain Jr. *(PI, Lecturer, University of Florida)* and Dr. Russell Jeter *(Consultant, Director of Analytics, Motus Nova)*

This project explores using artificial intelligence to transform the potential for a multiple-choice assessment tool to support individualized mathematics learning at the undergraduate level. Initiatives completed during the project include:

- A collection of approximately 80 multiple-choice question structures have been created that target students' procedural understanding of various topics in College Algebra;
- Qualitative and quantitative validation of multiple-choice question structures over 3 years of student data;
- Computer program named Automatic and Diagnostic Item Generation (Auto-DIG) was developed in tandem with the question structures to provide students with actionable feedback based on their answer choice.

By building on the current Auto-DIG software, this project aims to transform mathematics education by providing an approachable, open-source method for practitioners to integrate artificial intelligence and evidence-based research into their classroom to improve learning outcomes in mathematics courses. The project has produced two published papers (conference proceeding and peer-reviewed journal), another peer-reviewed article currently under review, five presentations at national conferences, and two invited talks. This project did not receive internal or external funding.

3.2 College Algebra Course Re-Design (2018-2019)

Project team: Dr. Darryl Chamberlain Jr. *(PI, Lecturer, University of Florida)*

This project sought to create a large-scale, mastery-based College Algebra course. Initiatives completed during the course of the re-design included:

- Reorganized content around functions as concepts to promote coherence and content retention;
- Developed open-source online homework with dynamic questions;
- Designed assessments that allowed students to repeat content until displaying mastery.

The re-design effort was viewed as a success by the UF Department of Mathematics chair and has continued after Dr. Chamberlain was hired by ERAU-W. This project has produced one journal article that is under review and one presentation at a national conference. This project was funded as part of Dr. Chamberlain's lecturer position at UF.

3.3 Online Resources for Prospective 8-12 Math Teachers (2020-present)

Project team: Dr. Catherine Paulocci *(PI, Assistant Professor, University of Florida)*, Dr. Darryl Chamberlain Jr. *(Co-PI, Lecturer, University of Florida)*, and Dr. Christopher Redding *(Co-PI, Assistant Professor, University of Florida)*

This project aims to support content knowledge development for secondary mathematics teachers, particularly those whose pathway to certification has included limited post-secondary studies of mathematics. Initially, it will focus on teachers in Florida who do not have a degree in mathematics or a relevant field and have earned temporary certification by taking the 8-12 Mathematics Subject Area Exam (SAE-Math). The project has collected resources available to secondary mathematics teachers who are preparing for certification and is analyzing these resources for coverage, conceptual levels, and validity. This project has produced one peer-reviewed journal article under review and one invited talk. This project received an internal UF grant of $29,923.

4 Budget Justification

Total Costs: $4069

Participant Incentives: $250
Interview participants will be compensated with a $25 gift card. Project will solicit for 10 participants.

Data Collection Expenses: $1,749
Professional transcription of each video interview to capture speech, mannerisms, gestures, and hand motions. Each interview will be approximately 60 minutes. Expense estimated for 600 minutes of video between two speakers at $2.75 per video minute with 6% tax.

Student Research Assistant: $1,920
2 months of support between January 2022 and March 2022 for student researcher to assist with quantitative and qualitative analysis of data. Specifically, student researcher will be used to establish inter-rater reliability for open coding. Quote at $12 per hour, 20 hours per week, for 8 weeks. A 3-month window to complete 8 weeks of work should provide sufficient flexibility to accommodate for any illness or other unexpected delays.

Software: $150
Access to Qualitative Analysis Software Dedoose for 7 months at $15 a month for Dr. Chamberlain and 3 months at $15 a month for unidentified student research assistant. While the student is paid for 8 weeks of work, this work may occur in 3 months if they become ill or have other unexpected delays.

2021-2022 WW Faculty Seed Grant Budget

Itemized Expense	Amount Requested
Participant Incentives	250
Professional Transcription	1,749
Student Research Assistant	1,920
Qualitative Analysis Software	150
Total	4,069

Project Title:

Developing Autonomous, Targeted Feedback in Precalculus

Investigator:

Dr. Darryl Chamberlain Jr.

Investigator Signature:

Darryl Chamberlain Jr. Digitally signed by Darryl Chamberlain Jr. Date: 2021.08.31 13:02:18 -04'00'

Department Chair:

Dr. Karen Keene

Department Chair Signature:

Karen Allen Keene Digitally signed by Karen Allen Keene Date: 2021.09.01 14:05:52 -04'00'

5 COVID-19 Pandemic Contingency Plan

All interactions with participants will occur digitally. There are no COVID-19 related risks to the proposed work.