Adapting the Argumentative Knowledge Construction Framework to Asynchronous Mathematical Discussions

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Weinberger and Fischer (2006) designed a framework for analyzing learning the context of asynchronous discussion activities. Operationalizing this framework, we analyzed the social and cognitive aspects of a discussion activity in an asynchronous calculus course. From this analysis, we identified aspects of Weinberger and Fischer's (2006) framework lacking in explanatory power for mathematics-specific discourse and developed an amended framework. We propose that this amended framework may enable in-depth analysis of major dimensions of students' mathematics knowledge construction as they engage in activities in an online asynchronous modality. This framework may also support the curriculum development for online asynchronous mathematics coursework.

Keywords: Calculus, Knowledge Co-Construction, Asynchronous Instruction, Design Research

Introduction

Classroom discourse can provide a powerful opportunity for students to gain mathematical knowledge. Though many students learn mathematics in asynchronous formats, the kinds of learning taking place in asynchronous formats remains severely underexamined. We are motivated to explore in more detail the learning within asynchronous discussions; specifically, we study discussion activities as we contend they are a primary point of contact in asynchronous formats among students, their peers, and their instructor.

In prior work (Reed, Chamberlain Jr., & Keene, 2022), we proposed use of Weinberger and Fischer's (2006) *Argumentative Knowledge Construction* framework for design research. We now present the results of a study operationalizing and extending Weinberger and Fischer's framework to examine the knowledge construction taking place in asynchronous calculus discussions. Our analysis of multiple discussion boards revealed that aspects of Weinberger and Fischer's framework required alteration to increase the explanatory power of the framework and enable an accounting of students' contributions to the discussion from an anti-deficit perspective. In this report, we present key aspects of our updated framework, highlighting its ability to gain insights into the learning taking place in discussions across social and cognitive dimensions. Accordingly, we propose to answer the following research questions:

1. How do the social, epistemic, and argumentative dimensions of knowledge construction interact with each other in the context of a mathematics classroom?
2. How does the *Argumentative Knowledge Construction* (AKC) Framework change when contextualized in a mathematics class?
3. What alterations to the AKC Framework increase our understanding of student learning in online calculus courses with discussions?

Theoretical Framework – Argumentative Knowledge Construction

Weinberger and Fischer proposed that computer-supported collaborative learning could be analyzed according to four dimensions: *participation, epistemic, argument,* and *social modes of co-construction* (Weinberger and Fischer, 2006).

The *participation dimension* examines the quantity and heterogeneity of students' contributions to the discussion board for each discussion activity. The *epistemic dimension* focuses on the content of students' contributions, attending particularly to the degree to which students' contributions adequately relate the particulars of a problem with the intended concepts that the problem engages. The *argument dimension* derives from Toulmin's (1964) model of arguments to qualify the types of micro- (single line) and macro- (multi-line) argumentative moves put forth by students in pursuit of a solution. Finally, the *dimensions of social modes of co-construction* "describe to what extent learners refer to contributions of their learning partners" (Weinberger & Fischer, 2006, p. 77). In an asynchronous modality, participants' textual, imagistic, and video submissions can be retroactively analyzed to build group-by-group comprehensive accounts of the knowledge construction associated with a particularly designed prompt. We consider the dimensions to productively account for knowledge construction in the online setting and so our work attempts to refine these dimensions in the mathematics context.

Table 1: Summary of Argumentative Knowledge Construction framework (Weinberger and Fischer, 2006).

Social Modes • Externalization • Elicitation • Quick Consensus Building • Integration-oriented Consensus Building • Conflict-oriented Consensus Building	**Epistemic** *Construction of…* • Problem Space • Conceptual Space • Problem ↔ Conceptual Space • Problem ↔ Prior Knowledge • Non-Epistemic Activities

Argument
• Single Arguments (micro-)
  o Simple, Qualified, Grounded, Grounded & Qualified, Non-Argumentative
• Line of Argumentation (macro-)
  o Argument, Counterargument, Integration, Non-Argumentative
	Participation
	• Quantity
	• Heterogeneity

Literature Review

Despite the prevalence of online mathematics courses, we still know very little about the ways that students make mathematical meaning in these formats (Trenholm et al., 2019). From what has been explored thus far, some have considered mathematics courses among the more difficult to teach in an online format (Engelbrecht & Harding, 2005).

While discussion activities are common in face-to-face mathematics courses, discussions are not widely used in asynchronous formats. For instance, in a survey targeting the format and content of asynchronous courses, 39% of instructors surveyed used at least 1 discussion (Trenholm et al., 2015). Thus far, research on discussion activities focusing on Weinberger and Fischer's (2016) AKC framework have studied face-to-face mathematics learning (Reed et al., 2022) and asynchronous learning in nonmathematical courses (Schrire, 2006; Clark & Sampson, 2008; Dubovi & Tabak, 2020). We contribute to this literature by examining the learning taking place within an asynchronous mathematics discussion activity, exploring how the AKC framework can be operationalized specifically in a mathematics setting.

As noted in the framework above, one part of the AKC framework is supported by the work of Toulmin (1958). In Toulmin's model, he provides a structure and function for argumentation in learning. This work involves the identification and connections between data used in argumentation, claims that participants make, and warrants they provide. Toulmin's model has been used in the analysis of in person mathematical discussions in active learning classrooms (Giannakoulias et al., 2010; Groth & Follmer, 2021; Mariotti & Pedemonte, 2019) but not in asynchronous learning. Nevertheless, this work supports our modifications of the AKC framework to contribute to our understanding and support design of new asynchronous learning activities.

Methodology

The data analyzed for this study were comprised of the textual records of two small-group discussion activities assigned to a fully asynchronous calculus course. Participants for this study were Calculus 1 students at an online, primarily undergraduate university during a 9-week term in spring 2022. The student population at this university is non-traditional: the average age is 31, 13.7% of the population is female, 60.3% of students are active military, and 19.5% of students are veterans. The data consisted of five discussions activities given to four assigned groups and moderated by one of the authors.

One researcher (the instructor of the course) initially reviewed each group's work according to the *participation* dimension. For our initial theory-building purposes, we sought to analyze groups for which there was potential to inform the interactions of the argumentative, social, and epistemic dimensions. As such, for the first analysis, we selected one group that we determined to have high participation, both in quantity and in heterogeneity. This group contained 3 male students that we refer to as BK, LG, and DG.

We began by examining this group's work on two discussion activities. For the first discussion activity, we individually coded each line of the discussion according to a single dimension of the AKC framework. We then met to compare and negotiate the codes for the selected dimension. After the discussion was coded for each dimension in the AKC framework, we discussed whether any lines in the discussion were not captured by the AKC framework or would benefit from a modification to the AKC framework. From this analysis, we proposed changes to that framework so that the epistemic, argumentative, and social dimensions yielded more robust insights for math-specific knowledge construction. We then analyzed the same group's discussion on a second discussion task according to this new framework in a similar iterative manner: individual coding for a single dimension, meeting to compare and negotiate codes, repeating these two steps until all dimensions were analyzed, and ending with a discussion of what was not captured by the modified AKC framework.

Due to space limitations, we will present results of the analysis on the second discussion prompt:

> *Your overall goal is to draw a random curve representing a function* f *defined on the interval* $[2, 12]$*, and then construct the graph of a second function* g *such that the following requirement is satisfied: For the composite function* $h(x) = g(f(x))$ *(meaning* g *composed with* f*),* $h'(x) = 2$ *at each* x*-value on* $[0, 10]$.

Data Analysis

Posts and replies to the discussion prompt were collected and manually entered to an Excel sheet. Posts were split by sentence and any equations/graphs were linked to a separate page. The researchers then individually coded each sentence by dimension according to the altered AKC framework. After each dimension was individually coded, the researchers negotiated the final coding for that dimension and discussed how effective the coding captured knowledge construction. Once all posts/replies were coded by each dimension, the researchers met to analyze the codes to gain cross-dimensional insights into the knowledge construction occurring during the discussion. This cross-dimensional analysis originally included tallies of the various codes and attention to the density or sparsity of certain interactions considering the progress made by each student in the epistemic dimension. While this provided general insights into how the students' interactions contributed to their epistemic progression, we determined that a more refined account of the social interactions would yield more robust insights. We present the results of our analysis considering this refined account of the social dimension.

Results

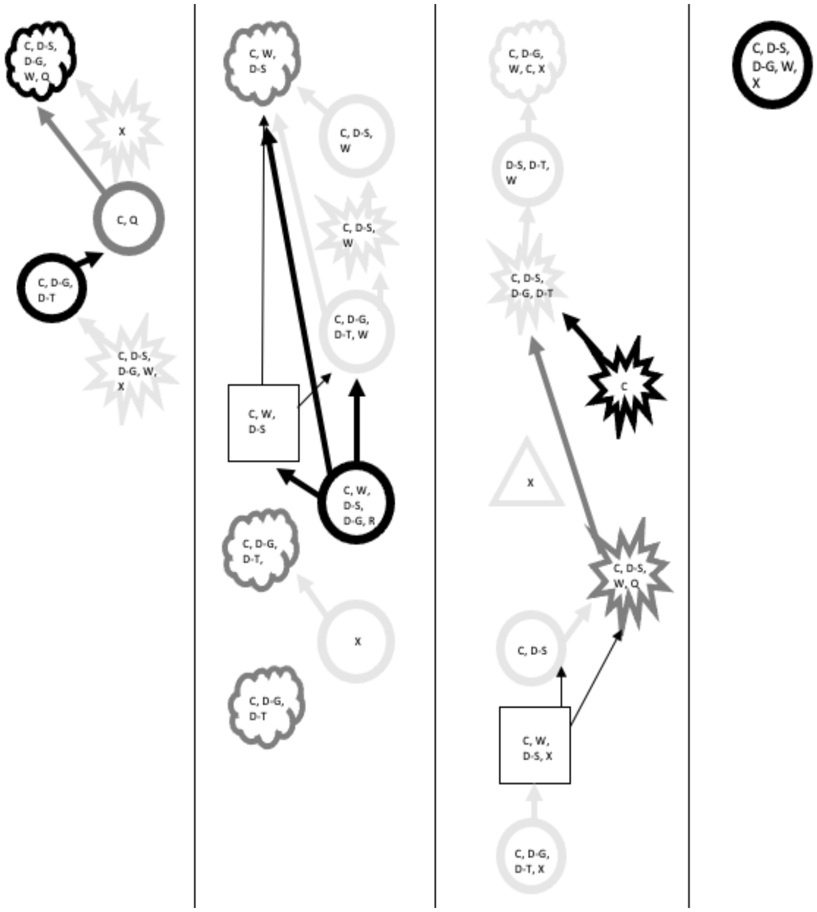
Due to space limitations for this proposal, we present one noteworthy result to appear from our initial analysis: conceptualizing the social dimension as a network of argumentative interactions. This alteration of the social and argumentative dimensions allowed us to analyze the interaction between macro- and micro-argumentative codes and provided insights that allowed for more readily seen inter-dimensional insights. What follows are two examples from the three-person group discussion of BK, LG, and DG. These examples will highlight what we learned by using the revised AKC framework. We will then present our revised framework in the discussion.

Interactions within the Argumentative Dimension

Figure 1 presents the visualization of 4 different chains of posts for the chosen discussion and group. The codes are chronological as they move down the page. Each column represents the analysis of one post and its replies. The three student participants were assigned different shades of grey: black (BK), light grey (LG), and dark grey (DG). Arrows represent replies to either the original post or another reply in the post chain. Visualizing these chains of arguments allowed us to identify patterns that were not easily identifiable in our original coarse analysis. Macro-argumentative codes are visualized by the shape of the post/reply as described in the legend.

We found that including Toulmin's model allowed for a more robust depiction of the social contributions of the students throughout the discussion than the original depiction of the social dimension. In these discussions, and more generally in mathematics discussions, social interactions largely (or most productively) entail argumentation. As such, including Toulmin's scheme in conjunction with the macro-argumentative codes allowed our new Argumentative dimension to account both for the argumentation taking place and for the social interactions contributing to knowledge construction. Moreover, the inclusion of Toulmin's Scheme for Argumentation allowed us to identify the type of data being used to make mathematical claims.

The visualization of the argumentative dimension also allowed for identifying social moves students made. For example, consider chain 1. We coded the first reply as a macro-counterargument and yet as a non-argumentative move. This captured a social counterargument that was not grounded in mathematics and was coded as conflict-oriented consensus building in the social dimension in the original AKC framework. We see another example in chain 2 with a circle X pairing, which suggests an integration-oriented consensus building in the social dimension in the original AKC framework. These results suggest that interactions within the argumentative dimension can account for codes in the original AKC social dimension, as we hypothesized by our alteration of the original framework.



Legend

Macro-Argumentative

Cloud	Argument
Star	Counterargument
Circle	Integration
Square	Instructor
Triangle	Non-Argumentative Move

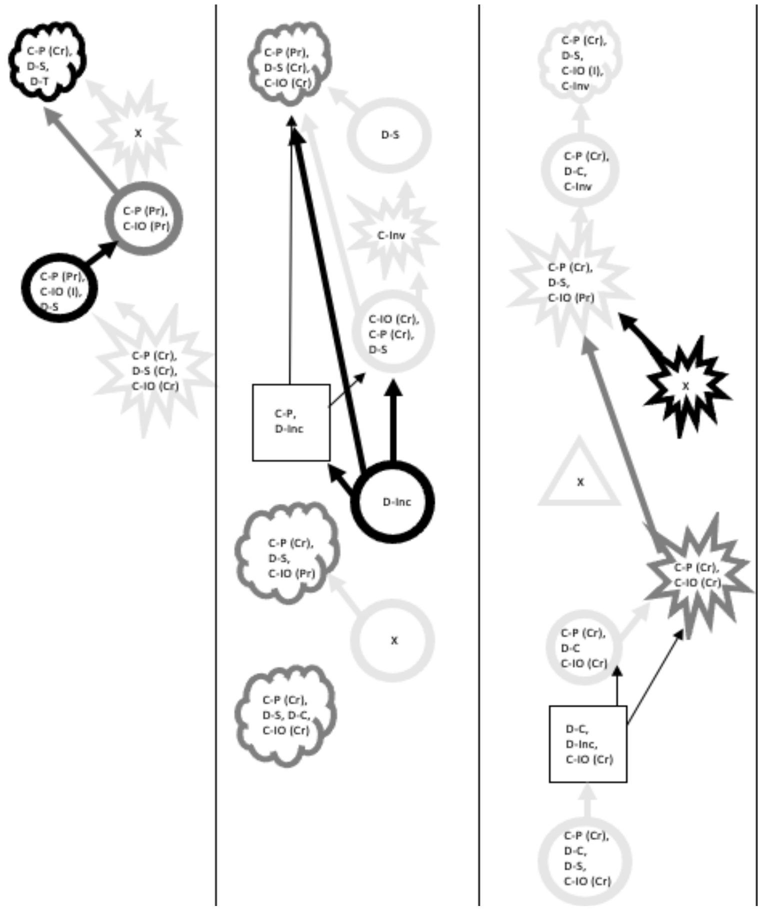
Micro-Argumentative

C	Claim
D-S D-G D-T	Data represented as Symbols, Graph, or Table
W	Warrant
Q	Modal Qualifier
R	Rebuttal
X	Non-Argumentative Move

Figure 1: Visualization of Macro- and Micro-Argumentative codes.

Interactions between the Argumentative and Epistemic Dimensions

Figure 2: Visualization of Macro-Argumentative and Epistemic dimensions. presents the visualization of the same 4 chains of posts with the epistemic codes placed within the macro-argument codes. The major result from our using our modified framework for the analysis is greater insight into the interactions between the epistemic and argumentative dimensions. Specifically, our original analysis flagged the original discussion as largely argument-driven (rather than incorporating counterarguments and integrations heavily), and that much of the epistemic change was independent of the social interactions in the discussion. With this altered framework, we drew clearer links between epistemic progression and counterarguments or integrations. For example, in chain 1 we see LG integrate BK's argument including composition as both a product (as BK did) and as inputs/outputs. We then see BK integrate LG's integration and include composition as inputs/outputs. Another example occurs in chain 2 where the instructor introduces the idea that derivatives describe where a function is increasing/decreasing and then BK uses this idea to integrate three different arguments. These examples illustrate that the inclusion of a new epistemic code can cause students to either counter this new idea or integrate it into their own argument. Moreover, tracking epistemic codes allows us to see that interactions are not restricted to explicit replies. For example, the conceptualization of derivative as increasing/decreasing, as presented by the instructor in chain 2, was present by BK in chain 4. This is important to note given the asynchronous nature of the discussion and the ability of students to read over what others have said without directly interacting.



Legend

Macro-Argumentative

Cloud	Argument
Star	Counterargument
Circle	Integration
Square	Instructor
Triangle	Non-Argumentative Move

Epistemic

C-P C-IO C-INV	Composition as Product, Input/Output, or Inverses
D-INC D-S D-RoC D-C D-T	Derivative as Increasing, Slope of Tangent, Rate of Change, Computation, or Tangent Line
I Pr Cr	Inattention, Progress, or Coordination

Figure 2: Visualization of Macro-Argumentative and Epistemic dimensions.

Discussion

After analyzing and discussing two sets of discussion posts, we arrived at three major revisions to Weinberger and Fischer's (2006) Argumentative Knowledge Construction framework:

- Inclusion of Toulmin's Scheme for micro-argumentative codes;
- Removal of the original social dimension; and
- Switch from deficit to asset-based view of epistemic dimension.

Toulmin's Scheme is included in the micro-argumentative codes as it provides a way to track the types of data students use in their arguments and the ways students justified their arguments. The original social dimension was removed as we found the macro- and micro-arguments sufficient for describing social moves students made when constructing knowledge. Moreover, the mathematical nature of the discussions tended toward non-social arguments. The switch from a deficit-based view of understanding as a connection between the problem and conceptual space to ways students understand concepts allows us to apply current mathematics education research to evaluate how students' understanding evolves over time. An overview of our current Argumentative Knowledge Construction framework specific to mathematics is presented in Table 2.

Table 2: Current Argumentative Knowledge Construction framework specific to mathematics.

<table>
<tr>
<td rowspan="2">
<u>Argumentation</u>
Macro-Arguments

Argument
Counterargument
Integration
Non-Argumentative Moves

Micro-Arguments

Claim
Data (Graphical, Symbolic, Tabular)
Modal Qualifier
Warrant, Backing, Rebuttal
Non-Argumentative Statements

</td>
<td>
<u>Epistemic</u>

Prompt-Specific Understandings
Progress Towards Coordination of Relevant Understandings
Broader Ways of Thinking Mathematically (When Applicable)

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<tr>
<td>
<u>Participation</u>

Quantity
Heterogeneity

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Conclusion and Next Steps

As is seen by looking at research published in undergraduate mathematics education, as well as the huge addition of online asynchronous mathematics teaching in the last 20 years, we believe that understanding how asynchronous learning may happen is a significant contribution to RUME. Our goal was to adapt Weinberger and Fischer's Argumentative Knowledge Construction Framework to asynchronous mathematics discussions and identify ways to systematically analyze student posts between dimensions. While more work is needed to refine the AKC framework specifically to mathematics, we feel that the use of macro-argumentative codes as a basis for visualizing student posts as a web of interactions will play an important role in analyzing knowledge development as it progresses through a discussion post.

We will continue to analyze further asynchronous discussions in Calculus using the modified framework. One particular type of response we would like to incorporate into the framework is non-content, non-social knowledge construction. For example, the instructor explaining how to use Desmos would not constitute mathematical knowledge in the epistemic dimension, nor would it constitute a social or non-social argument that would be captured in the argumentation dimension. Additionally, we anticipate expanding our work to other mathematics courses, and possibly sciences courses as well, which would require either generalizing what is contained in the epistemic dimension or creating subdimensions that work in unison.

This work is part of a larger design project to engage students and faculty in online mathematical learning. We will use our results and altered framework to support revisions and development of STEM courses in asynchronous environment. We also believe the new framework will be applicable to face-to-face student discussions in mathematics. Understanding how student communication is mathematics learning can be seen as an important area that can change mathematics instruction both online and in-person.

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